

- **Rule 2.** In a valid standard-form categorical syllogism, the middle term must be distributed in at least one premise.  
Violation: Fallacy of undistributed middle.
- **Rule 3.** In a valid standard-form categorical syllogism, if either term is distributed in the conclusion, then it must be distributed in the premises.  
Violation: Fallacy of the illicit major, or fallacy of the illicit minor.
- **Rule 4.** No standard-form categorical syllogism having two negative premises is valid.  
Violation: Fallacy of exclusive premises.
- **Rule 5.** If either premise of a valid standard-form categorical syllogism is negative, the conclusion must be negative.  
Violation: Fallacy of drawing an affirmative conclusion from a negative premise.
- **Rule 6.** No valid standard-form categorical syllogism with a particular conclusion can have two universal premises.  
Violation: Existential fallacy.

In Section 6.5, we presented an exposition of the fifteen valid forms of the categorical syllogism, identifying their moods and figures, and explaining their traditional Latin names:

AAA-1 (*Barbara*); EAE-1 (*Celarent*); AII-1 (*Darii*); EIO-1 (*Ferio*); AEE-2 (*Camestres*); EAE-2 (*Cesare*); AOO-2 (*Baroko*); EIO-2 (*Festino*); AII-3 (*Datisi*); IAI-3 (*Disamis*); EIO-3 (*Ferison*); OAO-3 (*Bokardo*); AEE-4 (*Camenes*); IAI-4 (*Dimaris*); EIO-4 (*Fresison*).

In the Appendix to Chapter 6 (which may be omitted), we presented the deduction of the fifteen valid forms of the categorical syllogism, demonstrating, through a process of elimination, that only those fifteen forms can avoid all violations of the six basic rules of the syllogism.

## Syllogisms in Ordinary Language

- 7.1 Syllogistic Arguments
- 7.2 Reducing the Number of Terms to Three
- 7.3 Translating Categorical Propositions into Standard Form
- 7.4 Uniform Translation
- 7.5 Enthymemes
- 7.6 Sorites
- 7.7 Disjunctive and Hypothetical Syllogisms
- 7.8 The Dilemma

### 7.1 Syllogistic Arguments

In ordinary discourse the arguments we encounter rarely appear as neatly packaged, standard-form categorical syllogisms. So the syllogistic arguments that arise in everyday speech cannot always be readily tested. They can be tested, however, if we *put* them into standard form—and we can generally do that by reformulating their constituent propositions. The term **syllogistic argument** refers to any argument that either *is* a standard-form categorical syllogism or that *can be reformulated as* a standard-form categorical syllogism without any loss or change of meaning.

We want to test the validity of syllogistic arguments. If they are fallacious or misleading, that will be most easily detected, as Immanuel Kant pointed out, when they are set out in correct syllogistic form. The process of reformulation is therefore important because the effective tests discussed in Chapter 6—Venn diagrams and the rules for categorical syllogisms—cannot be applied directly until the syllogism is in standard form. Putting it into standard form is called **reduction (or translation) to standard form**. When we reformulate (or reduce) a loosely put argument that appears in ordinary language into a classical syllogism, the resulting argument is called a **standard-form translation** of the original argument. Effecting this reformulation can present some difficulties.

## ■ EXAMPLE

1. Can any standard-form categorical syllogism be valid that contains exactly three terms, each of which is distributed in both of its occurrences?

## ■ SOLUTION

No, such a syllogism cannot be valid. If each of the three terms were distributed in both of its occurrences, all three of its propositions would have to be E propositions, and the mood of the syllogism would thus be EEE, which violates Rule 4, which forbids two negative premises.

2. In what mood or moods, if any, can a first-figure standard-form categorical syllogism with a particular conclusion be valid?
3. In what figure or figures, if any, can the premises of a valid standard-form categorical syllogism distribute both major and minor terms?
4. In what figure or figures, if any, can a valid standard-form categorical syllogism have two particular premises?
- \*5. In what figure or figures, if any, can a valid standard-form categorical syllogism have only one term distributed, and that one only once?
6. In what mood or moods, if any, can a valid standard-form categorical syllogism have just two terms distributed, each one twice?
7. In what mood or moods, if any, can a valid standard-form categorical syllogism have two affirmative premises and a negative conclusion?
8. In what figure or figures, if any, can a valid standard-form categorical syllogism have a particular premise and a universal conclusion?
9. In what mood or moods, if any, can a second figure standard-form categorical syllogism with a universal conclusion be valid?
- \*10. In what figure or figures, if any, can a valid standard-form categorical syllogism have its middle term distributed in both premises?
11. Can a valid standard-form categorical syllogism have a term distributed in a premise that appears undistributed in the conclusion?

## SUMMARY

In this chapter we have examined the standard-form categorical syllogism: its elements, its forms, its validity, and the rules governing its proper use.

In Section 6.1, the major, minor, and middle terms of a syllogism were identified:

- **Major term:** the predicate of the conclusion
- **Minor term:** the subject of the conclusion
- **Middle term:** the third term appearing in both premises but not in the conclusion.

We identified major and minor premises as those containing the major and minor terms, respectively. We specified that a categorical syllogism is in standard form when its propositions appear in precisely this order: major premise first, minor premise second, and conclusion last.

We also explained in Section 6.1 how the mood and figure of a syllogism are determined.

The mood of a syllogism is determined by the three letters identifying the types of its three propositions, A, E, I, or O. There are sixty-four possible different moods.

The figure of a syllogism is determined by the position of the middle term in its premises. The four possible figures are described and named thus:

- **First figure:** The middle term is the subject term of the major premise and the predicate term of the minor premise.  
Schematically:  $M-P, S-M, \text{ therefore } S-P.$
- **Second figure:** The middle term is the predicate term of both premises.  
Schematically:  $P-M, S-M, \text{ therefore } S-P.$
- **Third figure:** The middle term is the subject term of both premises.  
Schematically:  $M-P, M-S, \text{ therefore } S-P.$
- **Fourth figure:** The middle term is the predicate term of the major premise and the subject term of the minor premise.  
Schematically:  $P-M, M-S, \text{ therefore } S-P.$

In Section 6.2, we explained how the mood and figure of a standard-form categorical syllogism jointly determine its logical form. Because each of the sixty-four moods may appear in all four figures, there are exactly 256 standard-form categorical syllogisms, of which only a few are valid.

In Section 6.3, we explained the Venn diagram technique for testing the validity of syllogisms, using overlapping circles appropriately marked or shaded to exhibit the meaning of the premises.

In Section 6.4, we explained the six essential rules for standard-form syllogisms and named the fallacy that results when each of these rules is broken:

- **Rule 1.** A standard-form categorical syllogism must contain exactly three terms, each of which is used in the same sense throughout the argument.  
Violation: Fallacy of four terms.

**Cesare**

The traditional name for the valid syllogism with the mood and figure EAE-2

**Darii**

The traditional name for the valid syllogism with the mood and figure AII-1

**Datisi**

The traditional name for the valid syllogism with the mood and figure AII-3

**Disamis**

The traditional name for the valid syllogism with the mood and figure IAI-3

**Dimaris**

The traditional name for the valid syllogism with the mood and figure IAI-4

**Baroko**

The traditional name for the valid syllogism with the mood and figure AOO-2

**Ferio**

The traditional name for the valid syllogism with the mood and figure EIO-1

figure, EAE-1 (traditionally called *Celarent*), and the second figure, EAE-2 (traditionally called *Cesare*.)

**Summary of Case 2:** If the syllogism has an E conclusion, there are only four possibly valid forms: AEE-2, AEE-4, EAE-1, and EAE-2—*Camestres*, *Camenes*, *Celarent*, and *Cesare*, respectively.

**Case 3:** If the conclusion is an I proposition

In this case, neither premise can be an E or an O, because if either premise is negative, the conclusion must be negative. The two premises cannot both be A, because a syllogism with a particular conclusion cannot have two universal premises (Rule 6). Neither can both premises be I, because the middle term must be distributed in at least one premise (Rule 2). So the premises must be either AI or IA, and therefore the only possible moods with an I conclusion are AII and IAI.

AII is not possibly valid in the second figure or in the fourth figure because the middle term must be distributed in at least one premise. The only valid forms remaining for the mood AII, therefore, are AII-1 (traditionally called *Darii*) and AII-3 (traditionally called *Datisi*). If the mood is IAI, it cannot be IAI-1 or IAI-2, because they also would violate the rule that requires the middle term to be distributed in at least one premise. This leaves as valid only IAI-3 (traditionally called *Disamis*), and IAI-4 (traditionally called *Dimaris*).

**Summary of Case 3:** If the syllogism has an I conclusion, there are only four possibly valid forms: AII-1, AII-3, IAI-3, and IAI-4—*Darii*, *Datisi*, *Disamis*, and *Dimaris*, respectively.

**Case 4:** If the conclusion is an O proposition

In this case, the major premise cannot be an I proposition, because any term distributed in the conclusion must be distributed in the premises. So the major premise must be either an A or an E or an O proposition.

Suppose the major premise is an A. In that case, the minor premise cannot be either an A or an E, because two universal premises are not permitted when the conclusion (an O) is particular. Neither can the minor premise then be an I, because if it were, either the middle term would not be distributed at all (a violation of Rule 2), or a term distributed in the conclusion would not be distributed in the premises. So, if the major premise is an A, the minor premise has to be an O, yielding the mood AOO. In the fourth figure, AOO cannot possibly be valid, because in that case the middle term would not be distributed, and in the first figure and the third figure AOO cannot possibly be valid either, because that would result in terms being distributed in the conclusion

that were not distributed in the premises. For the mood AOO, the only possibly valid form remaining, if the major premise is an A, is therefore in the second figure, AOO-2 (traditionally called *Baroko*).

Now suppose (if the conclusion is an O) that the major premise is an E. In that case, the minor premise cannot be either an E or an O, because two negative premises are not permitted. Nor can the minor premise be an A, because two universal premises are precluded if the conclusion is particular (Rule 6). This leaves only the mood EIO—and this mood is valid in all four figures, traditionally known as *Ferio* (EIO-1), *Festino* (EIO-2), *Ferison* (EIO-3), and *Fresison* (EIO-4).

Finally, suppose (if the conclusion is an O) that the major premise is also an O proposition. Then, again, the minor premise cannot be an E or an O, because two negative premises are forbidden. And the minor premise cannot be an I, because then the middle term would not be distributed, or a term that is distributed in the conclusion would not be distributed in the premises. Therefore, if the major premise is an O, the minor premise must be an A, and the mood must be OAO. But OAO-1 is eliminated, because in that case the middle term would not be distributed. OAO-2 and OAO-4 are also eliminated, because in both a term distributed in the conclusion would then not be distributed in the premises. This leaves as valid only OAO-3 (traditionally known as *Bokardo*).

**Summary of Case 4:** If the syllogism has an O conclusion, there are only six possibly valid forms: AOO-2, EIO-1, EIO-2, EIO-3, EIO-4, and OAO-3—*Baroko*, *Ferio*, *Festino*, *Ferison*, *Fresison*, and *Bokardo*.

This analysis has demonstrated, by elimination, that there are exactly fifteen valid forms of the categorical syllogism: one if the conclusion is an A proposition, four if the conclusion is an E proposition, four if the conclusion is an I proposition, and six if the conclusion is an O proposition. Of these fifteen valid forms, four are in the first figure, four are in the second figure, four are in the third figure, and three are in the fourth figure. This completes the deduction of the fifteen valid forms of the standard-form categorical syllogism.

## EXERCISES

For students who enjoy the complexities of analytical syllogistics, here follow some theoretical questions whose answers can all be derived from the systematic application of the six rules of the syllogism set forth in Section 6.4. Answering these questions will be much easier if you have fully grasped the deduction of the fifteen valid syllogistic forms presented in this appendix.

**Festino**

The traditional name for the valid syllogism with the mood and figure EIO-2

**Ferison**

The traditional name for the valid syllogism with the mood and figure EIO-3

**Fresison**

The traditional name for the valid syllogism with the mood and figure EIO-4

**Bokardo**

The traditional name for the valid syllogism with the mood and figure OAO-3



## APPENDIX

## Deduction of the Fifteen Valid Forms of the Categorical Syllogism

In Section 6.5 the fifteen valid forms of the categorical syllogism were identified and precisely characterized. The unique name of each syllogism is also given there—a name assigned in view of its unique combination of mood and figure. The summary account of these fifteen syllogisms appears in the Overview immediately preceding.

It is possible to *prove* that these, and only these, are the valid forms of the categorical syllogism. This proof—the *deduction of the valid forms of the categorical syllogism*—is presented as an appendix, rather than in the body of the chapter, because mastering it is not essential for the student of logic. However, understanding it can give one a deeper appreciation of the *system* of syllogistics. And for those who derive satisfaction from the intricacies of analytical syllogistics, thinking through this deduction will be a pleasing, if somewhat arduous challenge.

We emphasize that if the chief aims of study are to recognize, understand, and apply the valid forms of the syllogism, as exhibited in Section 6.5, this appendix may be bypassed.

The deduction of the fifteen valid syllogisms is not easy to follow. Those who pursue it must keep two things very clearly in mind: (1) The *rules of the syllogism*, six basic rules set forth in Section 6.4, are the essential tools of the deduction; and (2) The *four figures of the syllogism*, as depicted in the Overview in Section 6.5 (p. 255) are referred to repeatedly as the rules are invoked.

We have seen that there are 256 *possible* forms of the syllogism, sixty-four moods (or combinations of the four categorical propositions) in each of the four figures. The deduction of the fifteen valid syllogisms proceeds by *eliminating* the syllogisms that violate one of the basic rules and that thus cannot be valid.

The conclusion of every syllogism is a categorical proposition, either *A*, or *E*, or *I*, or *O*. We begin by dividing all the possible syllogistic forms into four groups, each group having a conclusion with a different form (*A*, *E*, *I*, or *O*). Every syllogism must of course fall into one of these four groups. Taking each of the four groups in turn, we ask what characteristics a valid syllogism with such a conclusion must possess. That is, we ask what forms are *excluded* by one or more of the syllogistic rules if the conclusion is an *A* proposition, and if the conclusion is an *E* proposition, and so on.

After excluding all those invalid syllogisms, only the valid syllogisms remain. To assist in visualization, we note in the margin as we proceed the moods and figures, and the names, of the fifteen valid categorical syllogisms.

Case 1: If the conclusion of the syllogism is an *A* proposition

In this case, neither premise can be an *E* or an *O* proposition, because if either premise is negative, the conclusion must be negative (Rule 5). Therefore the two premises must be *I* or *A* propositions. The minor premise cannot be an *I* proposition because the minor term (the subject of the conclusion, which is an *A*) is distributed in the conclusion, and therefore if the minor premise were an *I* proposition, a term would be distributed in the conclusion that is not distributed in the premises, violating Rule 3. The two premises, major and minor, cannot be *I* and *A*, because if they were, either the distributed subject of the conclusion would not be distributed in the premise, violating Rule 3, or the middle term of the syllogism would not be distributed in either premise, violating Rule 2. So the two premises (if the conclusion is an *A*) must both be *A* as well, which means that the only possible valid mood is *AAA*. But in the second figure *AAA* again results in the middle term being distributed in neither premise; and in both the third figure and the fourth figure *AAA* results in a term being distributed in the conclusion that is not distributed in the premise in which it appears. Therefore, if the conclusion of the syllogism is an *A* proposition, the only valid form it can take is *AAA* in the first figure. This valid form, *AAA-1*, is the syllogism traditionally given the name *Barbara*.

**Summary of Case 1:** If the syllogism has an *A* conclusion, there is only one possibly valid form: *AAA-1—Barbara*.

*Barbara*  
The traditional name for the valid syllogism with the mood and figure *AAA-1*

Case 2: If the conclusion of the syllogism is an *E* proposition

Both the subject and the predicate of an *E* proposition are distributed, and therefore all three terms in the premises of a syllogism having such a conclusion must be distributed, and this is possible only if one of the premises is also an *E*. Both premises cannot be *E* propositions, because two negative premises are never allowed (Rule 4), and the other premise cannot be an *O* proposition because then both premises would also be negative. Nor can the other premise be an *I* proposition, for if it were, a term distributed in the conclusion would then not be distributed in the premise, violating Rule 3. So the other premise must be an *A*, and the two premises must be either *AE* or *EA*. The only possible moods (if the conclusion of the syllogism is an *E* proposition) are therefore *AEE* and *EAE*.

If the mood is *AEE*, it cannot be either in the first figure or in the third figure, because in either of those cases a term distributed in the conclusion would then not be distributed in the premises. Therefore, the mood *AEE* is possibly valid only in the second figure, *AEE-2* (traditionally called *Camestres*), or in the fourth figure, *AEE-4* (traditionally called *Camenes*). And if the mood is *EAE*, it cannot be in the third figure or in the fourth figure, because again that would mean that a term distributed in the conclusion would not be distributed in the premises, which leaves as valid only the first

*Camestres*  
The traditional name for the valid syllogism with the mood and figure *AEE-2*

*Camenes*  
The traditional name for the valid syllogism with the mood and figure *AEE-4*

*Celarent*  
The traditional name for the valid syllogism with the mood and figure *EAE-1*



of those syllogisms fully and to test their validity crisply, all rely on their being in standard form.\*

Classical logicians studied these forms closely, and they became fully familiar with their structure and their logical “feel.” This elegant system, finely honed, enabled reasoners confronting syllogisms in speech or in texts to recognize immediately those that were valid, and to detect with confidence those that were not. For centuries it was common practice to defend the solidity of reasoning in progress by giving the names of the forms of the valid syllogisms being relied on. The ability to provide these identifications even in the midst of heated oral disputes was considered a mark of learning and acumen, and it gave evidence that the chain of deductive reasoning being relied on was indeed unbroken. Once the theory of the syllogism has been fully mastered, this practical skill can be developed with profit and pleasure.

Syllogistic reasoning was so very widely employed, and so highly regarded as the most essential tool of scholarly argument, that the logical treatises of its original and greatest master, Aristotle, were venerated for more than a thousand years. His analytical account of the syllogism still carries the simple name that conveys respect and awe: the *Organon*, the *Instrument*.†

As students of this remarkable logical system, our proficiency in syllogistics may be only moderate—but we will nevertheless find it useful to have before us a synoptic account of all the valid syllogisms. These fifteen valid syllogisms (under the Boolean interpretation) may be divided by figure into four groups:‡

\*The burdensome consequences of ignoring standard form have been eloquently underscored by Keith Burgess-Jackson in his unpublished essay, “Why Standard Form Matters,” October 2003.

†Valid syllogisms are powerful weapons in controversy, but the effectiveness of those weapons depends, of course, on the truth of the premises. A great theologian, defiant in battling scholars who resisted his reform of the Church, wrote: “They may attack me with an army of six hundred syllogisms. . . .” (Erasmus, *The Praise of Folly*, 1511).

‡In the older tradition, in which reasoning from universal premises to particular conclusions was believed to be correct, the number of valid syllogisms (each uniquely named) was of course more than fifteen. To illustrate, if an I proposition may be inferred from its corresponding A proposition (as we think mistaken), the valid syllogism known as *Barbara* (AAA-1) will have a putatively valid “weakened” sister, *Barbari* (AAI-1); and if an O proposition may be inferred from its corresponding E proposition (as we think mistaken), the valid syllogism known as *Camestres* (AEE-2) will have a putatively valid “weakened” brother, *Camestrop* (AEO-2).

## OVERVIEW

### The Fifteen Valid Forms of the Standard-Form Categorical Syllogism

In the first figure (in which the middle term is the subject of the major premise and the predicate of the minor premise):

1. AAA-1 *Barbara*
2. EAE-1 *Celarent*
3. AII-1 *Darii*
4. EIO-1 *Ferio*

In the second figure (in which the middle term is the predicate of both premises):

5. AEE-2 *Camestres*
6. EAE-2 *Cesare*
7. AOO-2 *Baroko*
8. EIO-2 *Festino*

In the third figure (in which the middle term is the subject of both premises):

9. AII-3 *Datisi*
10. IAI-3 *Disamis*
11. EIO-3 *Ferison*
12. OAO-3 *Bokardo*

In the fourth figure (in which the middle term is the predicate of the major premise and the subject of the minor premise):

13. AEE-4 *Camenes*
14. IAI-4 *Dimaris*
15. EIO-4 *Fresison*

## EXERCISES

A. At the conclusion of Section 6.3, in exercise group B (on pp. 243–244), ten syllogisms were to be tested using Venn diagrams. Of these ten syllogisms, numbers 1, 4, 6, 9, and 10 are valid. What is the *name* of each of these five valid syllogisms?

### EXAMPLE

Number 1 is IAI-3 (*Disamis*).

It will be seen that:

- In the first figure the middle term is the subject of the major premise and the predicate of the minor premise;
- In the second figure the middle term is the predicate of both premises;
- In the third figure the middle term is the subject of both premises;
- In the fourth figure the middle term is the predicate of the major premise and the subject of the minor premise.

Each of the sixty-four moods can appear in each of the four figures. The mood and figure of a given syllogism, taken together, uniquely determine the logical form of that syllogism. Therefore there are (as noted earlier) exactly 256 ( $64 \times 4$ ) possible forms of the standard-form categorical syllogism.

The vast majority of these forms are not valid. We can eliminate every form that violates one or more of the syllogistic rules set forth in the preceding section. The forms that remain after this elimination are the only valid forms of the categorical syllogism. Of the 256 possible forms, there are exactly fifteen forms that cannot be eliminated and thus are valid.\*

To advance the mastery of syllogistics, classical logicians gave a unique name to every valid syllogism, each characterized completely by mood and figure. Understanding this small set of valid forms, and knowing the name of each, is very useful when putting syllogistic reasoning to work. Each name, carefully devised, contained three vowels representing (in standard-form order: major premise, minor premise, conclusion) the mood of the syllogism named. Where there are valid syllogisms of a given mood but in different figures, a unique name was assigned to each. Thus, for example, a syllogism of the mood EAE in the first figure was named *Celarent*, whereas

\*It should be borne in mind that we adopt here the Boolean interpretation of categorical propositions, according to which universal propositions (A and E propositions) do not have existential import. The classical interpretation of categorical propositions, according to which all the classes to which propositions refer do have members, makes acceptable some inferences that are found here to be invalid. Under that older interpretation, for example, it is plausible to infer the subaltern from its corresponding superaltern—to infer an I proposition from its corresponding A proposition, and an O proposition from its corresponding E proposition. This makes plausible the claim that there are other valid syllogisms (so-called weakened syllogisms) that are not considered valid here. Compelling reasons for the rejection of that older interpretation (and hence the justification of our stricter standards for valid syllogisms) were given at some length in Section 5.7.

a syllogism of the mood EAE in the second figure, also valid, was named *Cesare*.\*

These names had (and still have) a very practical purpose: If one knows that only certain combinations of mood and figure are valid, and can recognize by name those valid arguments, the merit of any syllogism in a given figure, or of a given mood, can be determined almost immediately. For example, the mood AOO is valid only in the second figure. That unique form (AOO-2) is known as *Baroko*.† One who is familiar with *Baroko* and able to discern it readily may be confident that a syllogism of this mood presented in any other figure may be rejected as invalid.

The standard form of the categorical syllogism is the key to the system. A neat and efficient method of identifying the few valid syllogisms from among the many possible syllogisms is at hand, but it depends on the assumption that the propositions of the syllogism in question either are in (or can be put into) standard order—major premise, minor premise, then conclusion. The unique identification of each valid syllogism relies on the specification of its mood, and its mood is determined by the letters characterizing its three constituent propositions *in that standard order*. If the premises of a valid syllogism were to be set forth in a different order, then that syllogism would remain valid, of course; the Venn diagram technique can prove this. But much would be lost. Our ability to identify syllogisms uniquely, and with that identification our ability to comprehend the forms

\*The principles that governed the construction of those traditional names, the selection and placement of consonants as well as vowels, were quite sophisticated. Some of these conventions relate to the place of the weakened syllogisms noted just above and are therefore not acceptable in the Boolean interpretation we adopt. Some other conventions remain acceptable. For example, the letter *s* that follows the vowel *e* indicates that when that E proposition is converted *simpliciter*, or simply (as all E propositions will convert), then that syllogism reduces to, or is transformed into, another syllogism of the same mood in the first figure, which is viewed as the most basic figure. To illustrate, *Festino*, in the second figure, reduces, when its major premise is converted simply, to *Ferio*; and *Cesare*, in the second figure, reduces to *Celarent*, and so on. The possibility of these and other reductions explains why the names of groups of syllogisms begin with the same consonant. The intricate details of the classical naming system need not be fully recounted here.

†Here is an example of *Baroko*:

All good mathematicians have creative intellects.

Some scholars do not have creative intellects.

Therefore some scholars are not good mathematicians.

With practice one comes to recognize the *cadence* of the different valid forms.

8. Some diamonds are not precious stones.  
Some carbon compounds are diamonds.  
 Therefore some carbon compounds are not precious stones.
9. All people who are most hungry are people who eat most.  
All people who eat least are people who are most hungry.  
 Therefore all people who eat least are people who eat most.
- \*10. Some spaniels are not good hunters.  
All spaniels are gentle dogs.  
 Therefore no gentle dogs are good hunters.

C. Identify the rule that is broken by any of the following syllogisms that are invalid, and name the fallacy that is committed.

#### EXAMPLE

1. All chocolate eclairs are fattening foods, because all chocolate eclairs are rich desserts, and some fattening foods are not rich desserts.

#### SOLUTION

In this syllogism the conclusion is affirmative ("all chocolate eclairs are fattening foods"), while one of the premises is negative ("some fattening foods are not rich desserts"). The syllogism therefore is invalid, violating the rule that if either premise is negative the conclusion must also be negative, thereby committing the fallacy of affirmative conclusion from a negative premise.

2. All inventors are people who see new patterns in familiar things, so all inventors are eccentrics, because all eccentrics are people who see new patterns in familiar things.
3. Some snakes are not dangerous animals, but all snakes are reptiles, therefore some dangerous animals are not reptiles.
4. Some foods that contain iron are toxic substances, for all fish containing mercury are foods that contain iron, and all fish containing mercury are toxic substances.
- \*5. All opponents of basic economic and political changes are outspoken critics of the liberal leaders of Congress, and all right-wing extremists are opponents of basic economic and political changes. It follows that all outspoken critics of the liberal leaders of Congress are right-wing extremists.
6. No writers of lewd and sensational articles are honest and decent citizens, but some journalists are not writers of lewd and sensational articles; consequently, some journalists are honest and decent citizens.

7. All supporters of popular government are democrats, so all supporters of popular government are opponents of the Republican Party, inasmuch as all Democrats are opponents of the Republican Party.
8. No coal-tar derivatives are nourishing foods, because all artificial dyes are coal-tar derivatives, and no artificial dyes are nourishing foods.
9. No coal-tar derivatives are nourishing foods, because no coal-tar derivatives are natural grain products, and all natural grain products are nourishing foods.
- \*10. All people who live in London are people who drink tea, and all people who drink tea are people who like it. We may conclude, then, that all people who live in London are people who like it.

## 6.5 Exposition of the Fifteen Valid Forms of the Categorical Syllogism

The *mood* of a syllogism is its character as determined by the forms (A, E, I, or O) of the three propositions it contains. There are sixty-four possible moods of the categorical syllogism; that is, sixty-four possible sets of three propositions: AAA, AAI, AAE, and so on, to . . . EOO, OOO.

The *figure* of a syllogism is its logical shape, as determined by the position of the middle term in its premises. So there are four possible figures, which can be most clearly grasped if one has in mind a chart, or iconic representation, of the four possibilities, as exhibited in the Overview table:

### OVERVIEW

The Four Figures				
	First Figure	Second Figure	Third Figure	Fourth Figure
Schematic Representation	$\begin{array}{c} M - P \\ \diagdown \\ S - M \end{array}$	$\begin{array}{c} P - M \\   \\ S - M \end{array}$	$\begin{array}{c} M - P \\   \\ M - S \end{array}$	$\begin{array}{c} P - M \\ \diagup \\ M - S \end{array}$
Description	$\therefore S - P$ The middle term is the subject of the major premise and the predicate of the minor premise.	$\therefore S - P$ The middle term is the predicate of both major and minor premises.	$\therefore S - P$ The middle term is the subject of both the major and minor premises.	$\therefore S - P$ The middle term is the predicate of the major premise and the subject of the minor premise.



**EXERCISES**

A. Identify the rule that is broken by invalid syllogisms of the following forms, and name the fallacy that each commits.

**EXAMPLE**

## 1. AAA-2

**SOLUTION**

Any syllogism in the second figure has the middle term as predicate of both the major and the minor premise. Thus any syllogism consisting of three A propositions, in the second figure, must read: All *P* is *M*; all *S* is *M*; therefore all *S* is *P*. *M* is not distributed in either of the premises in that form, and therefore it cannot validly be inferred from such premises that all *S* is *P*. Thus every syllogism of the form AAA-2 violates the rule that the middle term must be distributed in at least one premise, thereby committing the fallacy of the undistributed middle.

## 2. EAA-1

## 3. IAO-3

## 4. OEO-4

## \*5. AAA-3

## 6. IAI-2

## 7. OAA-3

## 8. EAO-4

## 9. OAI-3

## \*10. IEO-1

## 11. EAO-3

## 12. AII-2

## 13. EEE-1

## 14. OAO-2

## \*15. IAA-3

B. Identify the rule that is broken by any of the following syllogisms that are invalid, and name the fallacy that is committed.

**EXAMPLE**

1. All textbooks are books intended for careful study.

Some reference books are books intended for careful study.

Therefore some reference books are textbooks.

**SOLUTION**

In this syllogism, "textbooks" is the major term (the predicate of the conclusion) and "reference books" is the minor term (the subject of the conclusion). "Books intended for careful study" is therefore the middle term, and it appears as the predicate of both premises. In neither of the premises is this middle term distributed, so the syllogism violates the rule that the middle term must be distributed in at least one premise, thereby committing the fallacy of the undistributed middle.

2. All criminal actions are wicked deeds.

All prosecutions for murder are criminal actions.

Therefore all prosecutions for murder are wicked deeds.

3. No tragic actors are idiots.

Some comedians are not idiots.

Therefore some comedians are not tragic actors.

4. Some parrots are not pests.

All parrots are pets.

Therefore no pets are pests.

\*5. All perpetual motion devices are 100 percent efficient machines.

All 100 percent efficient machines are machines with frictionless bearings.

Therefore some machines with frictionless bearings are perpetual motion devices.

6. Some good actors are not powerful athletes.

All professional wrestlers are powerful athletes.

Therefore all professional wrestlers are good actors.

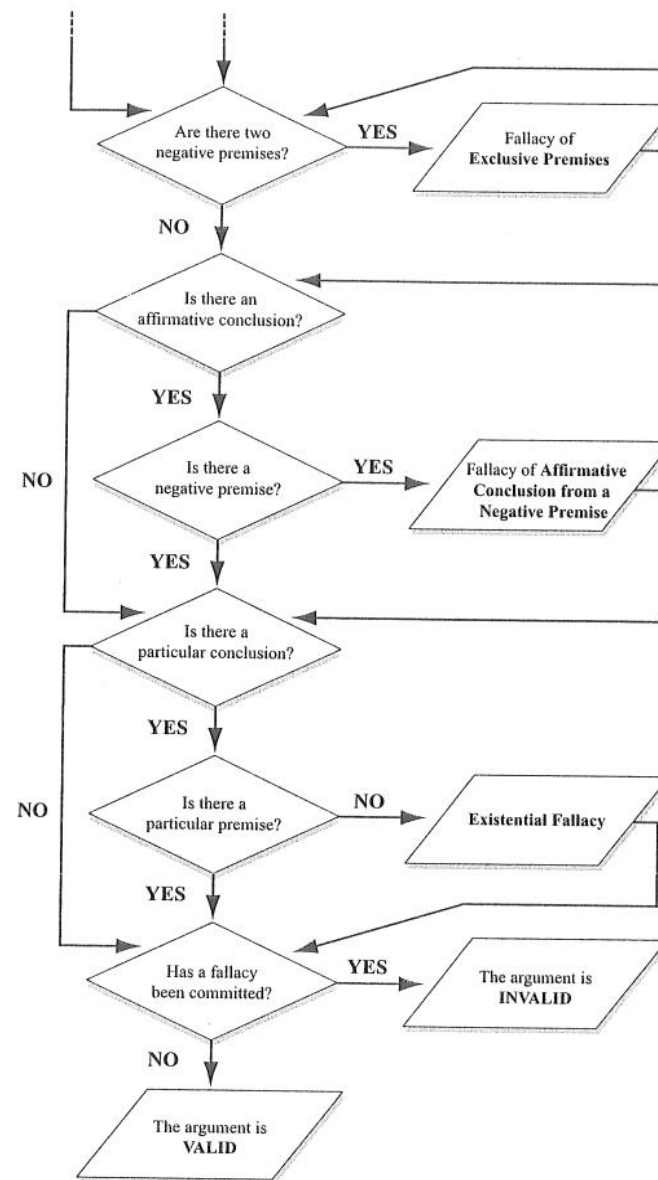
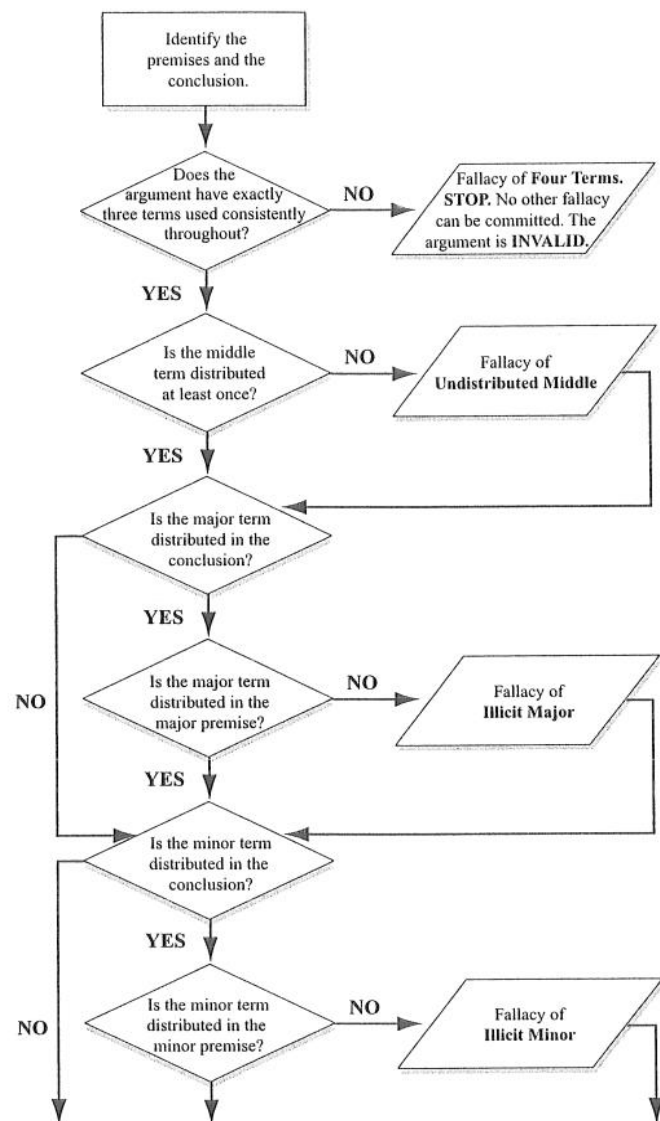
7. Some diamonds are precious stones.

Some carbon compounds are not diamonds.

Therefore some carbon compounds are not precious stones.

### FLOWCHART FOR APPLYING THE SIX SYLLOGISTIC RULES

The following chart captures the process for working through the six rules of validity for categorical syllogisms.



Adapted from Daniel E. Flage, *Essentials of Logic*, 2e (Englewood Cliffs, NJ: Prentice Hall, 1995)

is itself contained in the second. However, class inclusion can be stated only by affirmative propositions. Therefore, an affirmative conclusion can follow validly only from two affirmative premises. The mistake here is called the **fallacy of drawing an affirmative conclusion from a negative premise**.

If an affirmative conclusion requires two affirmative premises, as has just been shown, we can know with certainty that if either of the premises is negative, the conclusion must also be negative, or the argument is not valid.

Unlike some of the fallacies identified here, this fallacy is not common, because any argument that draws an affirmative conclusion from negative premises will be instantly recognized as highly implausible. Even an illustration of the mistake will appear strained:

No poets are accountants.

Some artists are poets.

Therefore some artists are accountants.

Immediately it will be seen that the exclusion of poets and accountants, asserted by the first premise of this syllogism, cannot justify any valid inference regarding the inclusion of artists and accountants.

**Rule 6.** From two universal premises no particular conclusion may be drawn. *In the Boolean interpretation of categorical propositions (explained in Section 5.7), universal propositions (A and E) have no existential import, but particular propositions (I and O) do have such import. Wherever the Boolean interpretation is supposed, as in this book, a rule is needed that precludes passage from premises that have no existential import to a conclusion that does have such import.*

This final rule is not needed in the traditional or Aristotelian account of the categorical syllogism, because that traditional account paid no attention to the problem of existential import. However, when existential import is carefully considered, it will be clear that if the premises of an argument do not assert the existence of anything at all, the conclusion will be unwarranted when, from it, the existence of some thing may be inferred. The mistake is called the **existential fallacy**.

Here is an example of a syllogism that commits this fallacy:

All household pets are domestic animals.

No unicorns are domestic animals.

Therefore some unicorns are not household pets.

If the conclusion of this argument were the universal proposition, "No unicorns are household pets," the syllogism would be perfectly valid for all. And because, under the traditional interpretation, existential import may be inferred from universal as well as from particular propositions, it would not be problematic

(in that traditional view) to say that the conclusion in the example given here is simply a "weaker" version of the conclusion we all agree is validly drawn.

In our Boolean view, however, the conclusion of the example ("Some unicorns are not household pets"), because it is a particular proposition, is not just "weaker," it is very different. It is an O proposition, a particular proposition, and thus has an existential import that the E proposition ("No unicorns are household pets") cannot have. Reasoning that is acceptable under the traditional view is therefore unacceptable under the Boolean view because, from the Boolean perspective, that reasoning commits the existential fallacy—a mistake that cannot be made under the traditional interpretation.\*

The six rules given here are intended to apply only to standard-form categorical syllogisms. In this realm they provide an adequate test for the validity of any argument. If a standard-form categorical syllogism violates any one of these rules, it is invalid; if it conforms to all of these rules, it is valid.

## OVERVIEW

Syllogistic Rules and Fallacies	
Rule	Associated Fallacy
1. Avoid four terms.	Four terms
2. Distribute the middle term in at least one premise.	Undistributed middle
3. Any term distributed in the conclusion must be distributed in the premises.	Illicit process of the major term (illicit major); Illicit process of the minor term (illicit minor)
4. Avoid two negative premises.	Exclusive premises
5. If either premise is negative, the conclusion must be negative.	Drawing an affirmative conclusion from a negative premise
6. No particular conclusion may be drawn from two universal premises.	Existential fallacy

\*Another interesting consequence of the difference between the traditional and the Boolean interpretation of categorical propositions is this: In the traditional view there is a need for a rule that states the converse of Rule 5 ("If either premise is negative, the conclusion must be negative"). The converse states simply that "If the conclusion of a valid syllogism is negative, at least one premise must be negative." And that is indisputable, because if the conclusion is negative, it denies inclusion. But affirmative premises assert inclusion. Therefore affirmative premises cannot entail a negative conclusion. This corollary is unnecessary in the Boolean interpretation because the rule precluding the existential fallacy (Rule 6) will, in the presence of the other rules, suffice to invalidate any syllogism with affirmative premises and a negative conclusion.



and neither does the second. Revolutionists is the middle term in this argument, and if the middle term is not distributed in at least one premise of a syllogism, that syllogism cannot be valid. The fallacy this syllogism commits is called the **fallacy of the undistributed middle**.

What underlies this rule is the need to *link* the minor and the major terms. If they are to be linked by the middle term, either the subject or the predicate of the conclusion must be related to the *whole* of the class designated by the middle term. If that is not so, it is possible that each of the terms in the conclusion may be connected to a different part of the middle term, and not necessarily connected with each other.

This is precisely what happens in the syllogism given in the preceding example. The Russians are included in a *part* of the class of revolutionists (by the first premise), and the anarchists are included in a *part* of the class of revolutionists (by the second premise)—but *different* parts of this class (the middle term of the syllogism) may be involved, and so the middle term does not successfully link the minor and major terms of the syllogism. In a valid syllogism, *the middle term must be distributed in at least one premise*.

**Rule 3.** Any term distributed in the conclusion must be distributed in the premises.

*To refer to all members of a class is to say more about that class than is said when only some of its members are referred to. Therefore, when the conclusion of a syllogism distributes a term that was undistributed in the premises, it says more about that term than the premises did. But a valid argument is one whose premises logically entail its conclusion, and for that to be true the conclusion must not assert any more than is asserted in the premises. A term that is distributed in the conclusion but is not distributed in the premises is therefore a sure mark that the conclusion has gone beyond its premises and has reached too far. The fallacy is that of illicit process.*

The conclusion may overreach with respect to either the minor term (its subject), or the major term (its predicate). So there are two different forms of illicit process, and different names have been given to the two formal fallacies involved. They are

Illicit process of the major term (an **illicit major**).

Illicit process of the minor term (an **illicit minor**).

To illustrate an illicit process of the major term, consider this syllogism:

All dogs are mammals.

No cats are dogs.

Therefore no cats are mammals.

The reasoning is obviously bad, but where is the mistake? The mistake is in the conclusion's assertion about *all* mammals, saying that all of them fall outside the class of cats. Bear in mind that an **A** proposition distributes its subject term but does not distribute its predicate term. Hence the premises make no assertion about *all* mammals—so the conclusion illicitly goes beyond what the premises assert. Because "mammals" is the major term in this syllogism, the fallacy here is that of an illicit major.

To illustrate illicit process of the minor term, consider this syllogism:

All traditionally religious people are fundamentalists.

All traditionally religious people are opponents of abortion.

Therefore all opponents of abortion are fundamentalists.

Again we sense quickly that something is wrong with this argument, and what is wrong is this: The conclusion makes an assertion about *all* opponents of abortion, but the premises make no such assertion; they say nothing about *all* abortion opponents. So the conclusion here goes illicitly beyond what the premises warrant. And in this case "opponents of abortion" is the minor term, so the fallacy is that of an illicit minor.

**Rule 4.** Avoid two negative premises.

*Any negative proposition (E or O) denies class inclusion; it asserts that some or all of one class is excluded from the whole of the other class. Two premises asserting such exclusion cannot yield the linkage that the conclusion asserts, and therefore cannot yield a valid argument. The mistake is named the **fallacy of exclusive premises**.*

Understanding the mistake identified here requires some reflection. Suppose we label the minor, major, and middle terms of the syllogism *S*, *P*, and *M*, respectively. What can two negative premises tell us about the relations of these three terms? They can tell us that *S* (the subject of the conclusion) is wholly or partially excluded from all or part of *M* (the middle term), and that *P* (the predicate of the conclusion) is wholly or partially excluded from all or part of *M*. However, any one of these relations may very well be established no matter how *S* and *P* are related. The negative premises cannot tell us that *S* and *P* are related by inclusion or by exclusion, partial or complete. Two negative premises (where *M* is a term in each) simply cannot justify the assertion of *any* relationship whatever between *S* and *P*. Therefore, if both premises of a syllogism are negative, the argument must be invalid.

**Rule 5.** If either premise is negative, the conclusion must be negative.

*If the conclusion is affirmative—that is, if it asserts that one of the two classes, *S* or *P*, is wholly or partly contained in the other—it can only be inferred from premises that assert the existence of a third class that contains the first and*

3. Some mammals are not horses, for no horses are centaurs, and all centaurs are mammals.
4. Some neurotics are not parasites, but all criminals are parasites; it follows that some neurotics are not criminals.
- \*5. All underwater craft are submarines; therefore no submarines are pleasure vessels, because no pleasure vessels are underwater craft.
6. No criminals were pioneers, for all criminals are unsavory persons, and no pioneers were unsavory persons.
7. No musicians are astronauts; all musicians are baseball fans; consequently, no astronauts are baseball fans.
8. Some Christians are not Methodists, for some Christians are not Protestants, and some Protestants are not Methodists.
9. No people whose primary interest is in winning elections are true liberals, and all active politicians are people whose primary interest is in winning elections, which entails that no true liberals are active politicians.
- \*10. No weaklings are labor leaders, because no weaklings are true liberals, and all labor leaders are true liberals.

## 6.4 Syllogistic Rules and Syllogistic Fallacies

A syllogism may fail to establish its conclusion in many different ways. To help avoid common errors we set forth rules—six of them—to guide the reasoner; any given standard-form syllogism can be evaluated by observing whether any one of these rules has been violated. Mastering the rules by which syllogisms may be evaluated also enriches our understanding of the syllogism itself; it helps us to see how syllogisms work, and to see why they fail to work if the rules are broken.

A violation of any one of these rules is a mistake, and it renders the syllogism invalid. Because it is a mistake of that special *kind*, we call it a fallacy; and because it is a mistake in the *form* of the argument, we call it a *formal fallacy* (to be contrasted with the *informal* fallacies described in Chapter 4). In reasoning with syllogisms, one must scrupulously avoid the fallacies that violations of the rules invariably yield. Each of these formal fallacies has a traditional name, explained below.

### Rule 1. Avoid four terms.

*A valid standard-form categorical syllogism must contain exactly three terms, each of which is used in the same sense throughout the argument.*

In every categorical syllogism, the conclusion asserts a relationship between two terms, the subject (minor term) and the predicate (major term). Such a conclusion

can be justified only if the premises assert the relationship of each of those two terms to the same third term (middle term). If the premises fail to do this consistently, the needed connection of the two terms in the conclusion cannot be established, and the argument fails. So every valid categorical syllogism must involve three terms—no more and no less. If more than three terms are involved, the syllogism is invalid. The fallacy thus committed is called the **fallacy of four terms**.

The mistake that commonly underlies this fallacy is equivocation, using one word or phrase with two different meanings. Most often it is the middle term whose meaning is thus shifted, in one direction to connect it with the minor term, in a different direction to connect it with the major term. In doing this the two terms of the conclusion are connected with two different terms (rather than with the same middle term), and so the relationship asserted by the conclusion is not established.\*

When the expression “*categorical syllogism*” was defined at the beginning of this chapter, we noted that by its nature every syllogism must have three and only three terms.<sup>†</sup> So this rule (“Avoid four terms”) may be regarded as a reminder to make sure that the argument being appraised really is a categorical syllogism.

### Rule 2. Distribute the middle term in at least one premise.

*A term is “distributed” in a proposition when (as was explained in Section 5.4) the proposition refers to all members of the class designated by that term. If the middle term is not distributed in at least one premise, the connection required by the conclusion cannot be made.*

Historian Barbara Tuchman (in *The Proud Tower*, New York: Macmillan, 1966) observed that many early critics of anarchism relied on the following “unconscious syllogism”:

All Russians were revolutionists.

All anarchists were revolutionists.

Therefore, all anarchists were Russians.

This syllogism is plainly invalid. Its mistake is that it asserts a connection between anarchists and Russians by relying on the links between each of those classes and the class of revolutionists—but revolutionists is an *undistributed* term in both of the premises. The first premise does not refer to all revolutionists,

\*Because it is the middle term that is most often manipulated, this fallacy is sometimes called “the fallacy of the ambiguous middle.” However, this name is not generally applicable, because one (or more) of the other terms may have its meaning shifted as well. Ambiguities may result in as many as five or six different terms being involved, but the mistake retains its traditional name: the fallacy of four terms.

<sup>†</sup>The term *syllogism* is sometimes defined more broadly than it has been in this book. The informal fallacy of equivocation, explained and warned against in Chapter 4, may arise in many different argumentative contexts, of course.

Examining Figure 6-9, we find that  $SP$  (which consists of the regions  $SPM$  and  $SP\bar{M}$ ) has been shaded out, so the syllogism's conclusion has already been diagrammed. How does this tell us that the given syllogism is valid? This syllogism concerns large classes of remote objects: There are many people whose attention is easily distracted while they are working, and they are scattered far and wide. However, we can construct a syllogism of the same form that involves objects that are immediately present and directly available for our inspection. These objects are the points within the unshaded portions of the circles labeled  $S$ ,  $P$ , and  $M$  in our Venn diagram. Here is the new syllogism:

All points within the unshaded part of the circle labeled  $P$  are points within the unshaded part of the circle labeled  $M$ .

No points within the unshaded part of the circle labeled  $M$  are points within the unshaded part of the circle labeled  $S$ .

Therefore no points within the unshaded part of the circle labeled  $S$  are points within the unshaded part of the circle labeled  $P$ .

This new syllogism refers to nothing remote; it is about the parts of a situation we ourselves have created: the Venn diagram we have drawn. All the parts and all the possibilities of inclusion and exclusion among these classes are immediately present to us and directly open to inspection. We can literally *see* all the possibilities here, and know that because all the points of  $P$  are also points of  $M$ , and because  $M$  and  $S$  have no points in common,  $S$  and  $P$  cannot possibly have any points in common. Because the new syllogism refers only to classes of points in the diagram, it can be literally *seen* to be valid by looking at the things it talks about. The original syllogism about classes of people has exactly the same form as this second one, so we are assured by the formal nature of syllogistic argument that the original syllogism is also valid. The explanation is exactly the same for Venn diagram proofs of the invalidity of invalid syllogisms; there, too, we test the original syllogism indirectly by testing directly a second syllogism that has exactly the same form and referring to the diagram that exhibits that form.

### EXERCISES

A. Write out each of the following syllogistic forms, using  $S$  and  $P$  as the subject and predicate terms of the conclusion, and  $M$  as the middle term. (Refer to the chart of the four syllogistic figures, if necessary, on p. 227.) Then test the validity of each syllogistic form using a Venn diagram.

#### EXAMPLE

1. AEE-1

### SOLUTION

We are told that this syllogism is in the first figure, and therefore the middle term,  $M$ , is the subject term of the major premise and the predicate term of the minor premise. (See chart on p. 228.) The conclusion of the syllogism is an E proposition and therefore reads: No  $S$  is  $P$ . The first (major) premise (which contains the predicate term of the conclusion) is an A proposition, and therefore reads: All  $M$  is  $P$ . The second (minor) premise (which contains the subject term of the conclusion) is an E proposition and therefore reads: No  $S$  is  $M$ . This syllogism therefore reads as follows:

All  $M$  is  $P$ .

No  $S$  is  $M$ .

Therefore no  $S$  is  $P$ .

Tested by means of a Venn diagram, as in Figure 6-10, this syllogism is shown to be invalid.

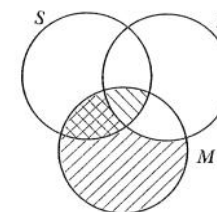


Figure 6-10

- |            |            |
|------------|------------|
| 2. EIO-2   | 3. OAO-3   |
| 4. AOO-4   | *5. EIO-4  |
| 6. OAO-2   | 7. AOO-1   |
| 8. EAE-3   | 9. EIO-3   |
| *10. IAI-4 | 11. AOO-3  |
| 12. EAE-1  | 13. IAI-1  |
| 14. OAO-4  | *15. EIO-1 |

B. Put each of the following syllogisms into standard form, name its mood and figure, and test its validity using a Venn diagram.

- \*1. Some reformers are fanatics, so some idealists are fanatics, because all reformers are idealists.
2. Some philosophers are mathematicians; hence some scientists are philosophers, because all scientists are mathematicians.



## VISUAL LOGIC

Where do I place the  $x$  in a Venn diagram?

In the Venn diagram representing a categorical syllogism, the three terms of the syllogism (Subject term, Predicate term, and Middle term) are represented by three interlocking circles labeled  $S$ ,  $P$ , and  $M$ .

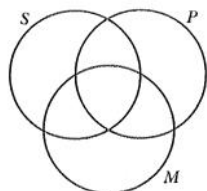


Diagram of three circles,  $S$ ,  $P$ , and  $M$ , with nothing else showing

When one of the premises of a syllogism calls for an  $x$  to be placed on a line in such a Venn diagram, we may ask: Which line? And why? Answer: The  $x$  is always placed on the line of the circle designating the class not mentioned in that premise.

**Example:** Suppose you are given as premise, "Some  $S$  is  $M$ ." You may not be able to determine whether the  $x$  representing that "some" is a  $P$  or is not a  $P$ —so the  $x$  goes on the line of the  $P$  circle, thus:

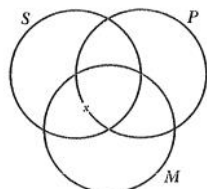


Diagram of three circles with  $x$  on the  $P$  circle

**Another example:** Suppose you are given as premise, "Some  $M$  is not  $P$ ." You may not be able to determine whether the  $M$  that is not  $P$  is an  $S$  or is not an  $S$ —so the  $x$  goes on the line of the  $S$  circle, thus:

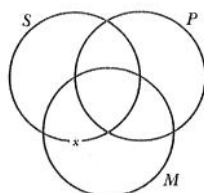


Diagram of three circles with  $x$  on the  $S$  circle

or great scientists, or sodium salts. The relations of inclusion or exclusion among such classes may be reasoned about and may be empirically discoverable in the course of scientific investigation. But they certainly are not open to direct inspection, because not all members of the classes involved are ever present at one time to be inspected. We can, however, examine situations of our own making, in which the only classes concerned contain by their very definitions only things that are present and open to direct inspection. And we can argue syllogistically about such situations of our own making. Venn diagrams are devices for expressing standard-form categorical propositions, but they also are situations of our own making, patterns of graphite or ink on paper, or lines of chalk on blackboards. And the propositions they express can be interpreted as referring to the diagrams themselves. An example can help to make this clear. Suppose we have a particular syllogism whose terms denote various kinds of people who are successful, interested in their work, and able to concentrate, and who may be scattered widely over all parts of the world:

All successful people are people who are keenly interested in their work.

No people who are keenly interested in their work are people whose attention is easily distracted when they are working.

Therefore no people whose attention is easily distracted when they are working are successful people.

Its form is AEE-4, and it may be schematized as

All  $P$  is  $M$ .

No  $M$  is  $S$ .

$\therefore$  No  $S$  is  $P$ .

We may test it by constructing the Venn diagram shown in Figure 6-9, in which regions  $SP\bar{M}$  and  $\bar{S}P\bar{M}$  are shaded out to express the first premise, and  $S\bar{P}M$  and  $SPM$  are shaded out to express the second premise.

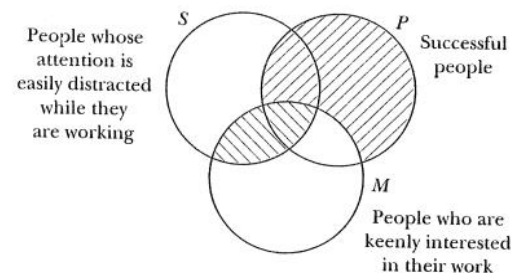


Figure 6-9

part of the circles labeled "Paupers" and "Egotists." This overlapping part consists of both of the regions  $SP\bar{M}$  and  $SPM$ , which together constitute  $SP$ . There is an  $x$  in the region  $SPM$ , so there is an  $x$  in the overlapping part  $SP$ . What the conclusion of the syllogism says has already been diagrammed by the diagramming of its premises; therefore the syllogism is valid.

Let us consider still another example, the discussion of which will bring out another important point about the use of Venn diagrams. Let's say we are testing the argument

All great scientists are college graduates.

Some professional athletes are college graduates.

Therefore some professional athletes are great scientists.

After diagramming the universal premise first (Figure 6-7) by shading out both regions  $SP\bar{M}$  and  $SPM$ ,

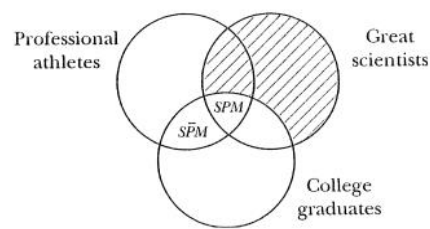


Figure 6-7

we may still be puzzled about where to put the  $x$  needed in order to diagram the particular premise. That premise is "Some professional athletes are college graduates," so an  $x$  must be inserted somewhere in the overlapping part of the two circles labeled "Professional athletes" and "College graduates." That overlapping part, however, contains two regions,  $SPM$  and  $SP\bar{M}$ . In which of these should we put an  $x$ ? The premises do not tell us, and if we make an arbitrary decision to place it in one rather than the other, we would be inserting more information into the diagram than the premises warrant—which would spoil the diagram's use as a test for validity. Placing  $x$ 's in each of them would also go beyond what the premises assert. Yet by placing an  $x$  on the line that divides the overlapping region  $SM$  into the two parts  $SPM$  and  $SP\bar{M}$ , we can diagram exactly what the second premise asserts without adding anything to it. Placing an  $x$  on the line between two regions indicates that there is something that belongs in one of them, but does not indicate which one. The completed diagram of both premises thus looks like Figure 6-8.

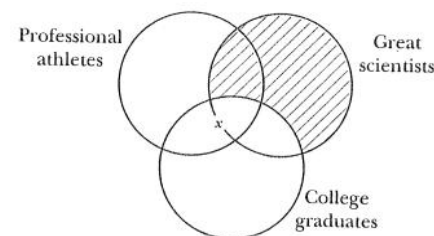


Figure 6-8

When we inspect this diagram of the premises to see whether the conclusion of the syllogism has already been diagrammed in it, we find that it has not. For the conclusion, "Some professional athletes are great scientists," to be diagrammed, an  $x$  must appear in the overlapping part of the two upper circles, either in  $SP\bar{M}$  or in  $SPM$ . The first of these is shaded out and certainly contains no  $x$ . The diagram does not show an  $x$  in  $SPM$  either. True, there must be a member of either  $SPM$  or  $SP\bar{M}$ , but the diagram does not tell us that it is in the former rather than the latter and so, for all the premises tell us, the conclusion may be false. We do not know that the conclusion is false, only that it is not asserted or implied by the premises. The latter is enough, however, to let us know that the argument is invalid. The diagram suffices to show not only that the given syllogism is invalid, but that *all* syllogisms of the form AII-2 are invalid.

The general technique of using Venn diagrams to test the validity of any standard-form syllogism may be summarized as follows. First, label the circles of a three-circle Venn diagram with the syllogism's three terms. Next, diagram both premises, diagramming the universal one first if there is one universal and one particular, being careful in diagramming a particular proposition to put an  $x$  on a line if the premises do not determine on which side of the line it should go. Finally, inspect the diagram to see whether the diagram of the premises contains a diagram of the conclusion: If it does, the syllogism is valid; if it does not, the syllogism is invalid.

What is the theoretical rationale for using Venn diagrams to distinguish valid from invalid syllogisms? The answer to this question divides into two parts. The first part has to do with the formal nature of syllogistic argument as explained in Section 6.2. It was shown there that one legitimate test of the validity or invalidity of a syllogism is to establish the validity or invalidity of a different syllogism that has exactly the same form. This technique is basic to the use of Venn diagrams.

The explanation of *how* the diagrams serve this purpose constitutes the second part of the answer to our question. Ordinarily, a syllogism will be about classes of objects that are not all present, such as the class of all musicians,

This is the diagram for both premises of the syllogism AAA-1:

All *M* is *P*.

All *S* is *M*.

∴ All *S* is *P*.

This syllogism is valid if and only if the two premises imply or entail the conclusion—that is, if together they say what is said by the conclusion. Consequently, diagramming the premises of a valid argument should suffice to diagram its conclusion also, with no further marking of the circles needed. To diagram the conclusion “All *S* is *P*” is to shade out both the portion labeled  $\bar{S}\bar{P}M$  and the portion labeled  $\bar{S}PM$ . Inspecting the diagram that represents the two premises, we see that it also diagrams the conclusion. And from this we can conclude that AAA-1 is a valid syllogism.\*

Let us now apply the Venn diagram test to an obviously invalid syllogism, one containing three A propositions in the second figure:

All dogs are mammals.

All cats are mammals.

Therefore all cats are dogs.

Diagramming both premises gives us Figure 6-5.

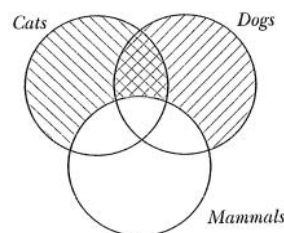


Figure 6-5

In this diagram, where *S* designates the class of all cats, *P* the class of all dogs, and *M* the class of all mammals, the portions  $\bar{S}\bar{P}M$ ,  $\bar{S}PM$ , and  $\bar{S}\bar{P}\bar{M}$ , have been

\*The mood of this syllogism is AAA because it consists of three A propositions; it is in the first figure because its middle term is the subject of its major premise and the predicate of its minor premise. Any syllogism of this valid form, AAA-1, is called (as noted earlier) a syllogism in Barbara. The names of other valid syllogisms will be given in Section 6.5.

shaded out. But the conclusion has not been diagrammed, because the part  $\bar{S}PM$  has been left unshaded, and to diagram the conclusion both  $\bar{S}\bar{P}M$  and  $\bar{S}PM$  must be shaded. Thus we see that diagramming both the premises of a syllogism of form AAA-2 does *not* suffice to diagram its conclusion, which proves that the conclusion says something more than is said by the premises, which shows that the premises do not imply the conclusion. An argument whose premises do not imply its conclusion is invalid, so our diagram proves that the given syllogism is invalid. (It proves, in fact, that *any* syllogism of the form AAA-2 is invalid.)

When we use a Venn diagram to test a syllogism with one universal premise and one particular premise, it is important to *diagram the universal premise first*. Thus, in testing the AII-3 syllogism,

All artists are egotists.

Some artists are paupers.

Therefore some paupers are egotists.

we should diagram the universal premise, “All artists are egotists,” before inserting an *x* to diagram the particular premise, “Some artists are paupers.” Properly diagrammed, the syllogism looks like Figure 6-6.

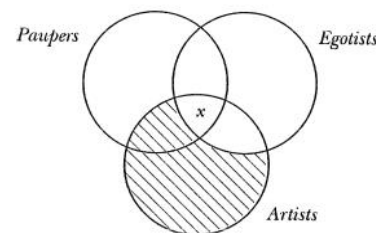


Figure 6-6

Had we tried to diagram the particular premise first, before the region  $\bar{S}PM$  was shaded out along with  $\bar{S}\bar{P}M$  in diagramming the universal premise, we would not have known whether to insert an *x* in  $SPM$  or in  $\bar{S}PM$  or in both. And had we put it in  $\bar{S}PM$  or on the line separating it from  $SPM$ , the subsequent shading of  $\bar{S}PM$  would have obscured the information the diagram was intended to exhibit. Now that the information contained in the premises has been inserted into the diagram, we can examine it to see whether the conclusion already has been diagrammed. If the conclusion, “Some paupers are egotists,” has been diagrammed, there will be an *x* somewhere in the overlapping



We first draw two circles, just as we did to diagram a single proposition, and then we draw a third circle beneath, overlapping both of the first two. We label the three circles  $S$ ,  $P$ , and  $M$ , in that order. Just as one circle labeled  $S$  diagrammed both the class  $S$  and the class  $\bar{S}$ , and as two overlapping circles labeled  $S$  and  $P$  diagrammed four classes ( $SP$ ,  $S\bar{P}$ ,  $\bar{S}P$ , and  $\bar{S}\bar{P}$ ), three overlapping circles, labeled  $S$ ,  $P$ , and  $M$ , diagram eight classes:  $S\bar{P}\bar{M}$ ,  $SP\bar{M}$ ,  $\bar{S}P\bar{M}$ ,  $S\bar{P}M$ ,  $SPM$ ,  $\bar{S}PM$ ,  $\bar{S}\bar{P}M$ , and  $\bar{S}\bar{P}\bar{M}$ . These are represented by the eight parts into which the three circles divide the plane, as shown in Figure 6-1.

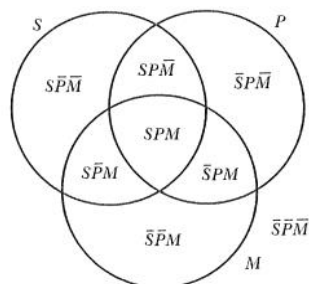


Figure 6-1

Figure 6-1 can be interpreted, for example, in terms of the various different classes determined by the class of all Swedes ( $S$ ), the class of all peasants ( $P$ ), and the class of all musicians ( $M$ ).  $SPM$  is the product of these three classes, which is the class of all Swedish peasant musicians.  $SP\bar{M}$  is the product of the first two and the complement of the third, which is the class of all Swedish peasants who are not musicians.  $S\bar{P}M$  is the product of the first and third and the complement of the second: the class of all Swedish musicians who are not peasants.  $\bar{S}P\bar{M}$  is the product of the second and third classes with the complement of the first: the class of all peasant musicians who are not Swedes.  $\bar{S}PM$  is the product of the second class with the complements of the other two: the class of all peasants who are neither Swedes nor musicians.  $\bar{S}\bar{P}M$  is the product of the third class and the complements of the first two: the class of all musicians who are neither Swedes nor peasants. Finally,  $\bar{S}\bar{P}\bar{M}$  is the product of the complements of the three original classes: the class of all things that are neither Swedes nor peasants nor musicians.

If we focus our attention on just the two circles labeled  $P$  and  $M$ , it is clear that by shading out, or by inserting an  $x$ , we can diagram any standard-form categorical proposition whose two terms are  $P$  and  $M$ , regardless of which is the subject term and which is the predicate. Thus, to diagram the proposition "All  $M$  is  $P$ " ( $M\bar{P} = 0$ ), we shade out all of  $M$  that is not contained

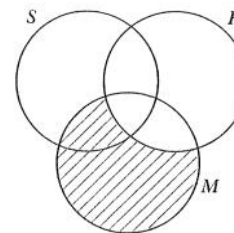


Figure 6-2

in (or overlapped by)  $P$ . This area, it is seen, includes both the portions labeled  $S\bar{P}\bar{M}$  and  $\bar{S}\bar{P}\bar{M}$ . The diagram then becomes Figure 6-2.

If we focus our attention on just the two circles  $S$  and  $M$ , by shading out, or by inserting an  $x$ , we can diagram any standard-form categorical proposition whose terms are  $S$  and  $M$ , regardless of the order in which they appear in it. To diagram the proposition "All  $S$  is  $M$ " ( $S\bar{M} = 0$ ), we shade out all of  $S$  that is not contained in (or overlapped by)  $M$ . This area, it is seen, includes both the portions labeled  $S\bar{P}\bar{M}$  and  $S\bar{P}M$ . The diagram for this proposition will appear as Figure 6-3.

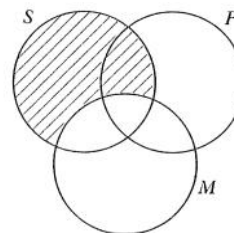


Figure 6-3

The advantage of using three overlapping circles is that it allows us to diagram two propositions together—on the condition, of course, that only three different terms occur in them. Thus diagramming both "All  $M$  is  $P$ " and "All  $S$  is  $M$ " at the same time gives us Figure 6-4.

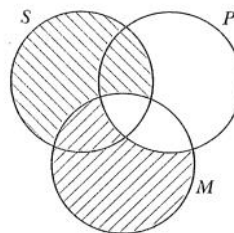


Figure 6-4

pattern as this analogous one about horses and rabbits. This one is invalid—so *your* argument is invalid. This is an excellent method of arguing; the logical analogy is one of the most powerful weapons that can be used in debate.

Underlying the method of logical analogy is the fact that the validity or invalidity of such arguments as the categorical syllogism is a purely formal matter. Any fallacious argument can be proved to be invalid by finding a second argument that has exactly the same form and is known to be invalid by the fact that its premises are known to be true while its conclusion is known to be false. (It should be remembered that an invalid argument may very well have a true conclusion—that an argument is invalid simply means that its conclusion is not logically implied or necessitated by its premises.)

This method of testing the validity of arguments has serious limitations, however. Sometimes a logical analogy is difficult to “think up” on the spur of the moment. And there are far too many invalid forms of syllogistic argument (well over two hundred!) for us to prepare and remember refuting analogies of each of them in advance. Moreover, although being able to think of a logical analogy with true premises and false conclusion proves its form to be invalid, *not* being able to think of one does not prove the form valid, for it may merely reflect the limitations of our thinking. There may be an invalidating analogy even though we are not able to think of it. A more effective method of establishing the formal validity or invalidity of syllogisms is required. The explanation of effective methods of testing syllogisms is the object of the remaining sections of this chapter.

### EXERCISES

Refute any of the following arguments that are invalid by the method of constructing logical analogies.

#### EXAMPLE

1. All business executives are active opponents of increased corporation taxes, for all active opponents of increased corporation taxes are members of the chamber of commerce, and all members of the chamber of commerce are business executives.

#### SOLUTION

One possible refuting analogy is this: All bipeds are astronauts, for all astronauts are humans and all humans are bipeds.

2. No medicines that can be purchased without a doctor's prescription are habit-forming drugs, so some narcotics are not habit-forming drugs, because some narcotics are medicines that can be purchased without a doctor's prescription.
3. No Republicans are Democrats, so some Democrats are wealthy stockbrokers, because some wealthy stockbrokers are not Republicans.
4. No college graduates are persons having an IQ of less than 70, but all persons who have an IQ of less than 70 are morons, so no college graduates are morons.
- \*5. All fireproof buildings are structures that can be insured at special rates, so some structures that can be insured at special rates are not wooden houses, because no wooden houses are fireproof buildings.
6. All blue-chip securities are safe investments, so some stocks that pay a generous dividend are safe investments, because some blue-chip securities are stocks that pay a generous dividend.
7. Some pediatricians are not specialists in surgery, so some general practitioners are not pediatricians, because some general practitioners are not specialists in surgery.
8. No intellectuals are successful politicians, because no shy and retiring people are successful politicians, and some intellectuals are shy and retiring people.
9. All trade union executives are labor leaders, so some labor leaders are conservatives in politics, because some conservatives in politics are trade union executives.
- \*10. All new automobiles are economical means of transportation, and all new automobiles are status symbols; therefore some economical means of transportation are status symbols.

### 6.3 Venn Diagram Technique for Testing Syllogisms

In Chapter 5 we explained the use of two-circle Venn diagrams to represent standard-form categorical propositions. In order to test a categorical syllogism using Venn diagrams, one must first represent both of its premises in one diagram. That requires drawing *three* overlapping circles, for the two premises of a standard-form syllogism contain three different terms—minor term, major term, and middle term—which we abbreviate as *S*, *P*, and *M*, respectively.

6. All CD players are delicate mechanisms, but no delicate mechanisms are suitable toys for children; consequently, no CD players are suitable toys for children.
7. All juvenile delinquents are maladjusted individuals, and some juvenile delinquents are products of broken homes; hence some maladjusted individuals are products of broken homes.
8. No stubborn individuals who never admit a mistake are good teachers, so, because some well-informed people are stubborn individuals who never admit a mistake, some good teachers are not well-informed people.
9. All proteins are organic compounds, hence all enzymes are proteins, as all enzymes are organic compounds.
- \*10. No sports cars are vehicles intended to be driven at moderate speeds, but all automobiles designed for family use are vehicles intended to be driven at moderate speeds, from which it follows that no sports cars are automobiles designed for family use.

## 6.2 The Formal Nature of Syllogistic Argument

In all deductive logic we aim to discriminate valid arguments from invalid ones; in classical logic this becomes the task of discriminating valid syllogisms from invalid ones. It is reasonable to assume that the constituent propositions of a syllogism are all contingent—that is, that no one of those propositions is necessarily true, or necessarily false. Under this assumption, the validity or invalidity of any syllogism depends entirely on its *form*. Validity and invalidity are completely independent of the specific content of the argument or its subject matter. Thus any syllogism of the form AAA-1

All M is P  
All S is M  
 ∴ All S is P

is valid, regardless of its subject matter. The name of this syllogism's form is *Barbara*; no matter what terms are substituted for the letters S, P, and M, the resulting argument, "in *Barbara*," will always be valid. If we substitute "Athenians" and "humans" for S and P, and "Greeks" for M, we obtain this valid argument:

All Greeks are humans.  
All Athenians are Greeks.  
 ∴ All Athenians are humans.

And if we substitute the terms "soaps," "water-soluble substances," and "sodium salts" for the letters S, P, and M in the same form, we obtain

All sodium salts are water-soluble substances.  
All soaps are sodium salts.  
 Therefore all soaps are water-soluble substances.

which also is valid.

A valid syllogism is a formally valid argument—valid by virtue of its form alone. This implies that if a given syllogism is valid, *any other syllogism of the same form will also be valid*. And if a syllogism is invalid, *any other syllogism of the same form will also be invalid*.<sup>\*</sup> The common recognition of this fact is attested to by the frequent use of "logical analogies" in argumentation. Suppose that we are presented with the argument

All liberals are proponents of national health insurance.  
Some members of the administration are proponents of national health insurance.  
 Therefore some members of the administration are liberals.

and felt (justifiably) that, regardless of the truth or falsehood of its constituent propositions, the argument is invalid. The best way to expose its fallacious character is to construct another argument that has exactly the same form but whose invalidity is immediately apparent. We might seek to expose the given argument by replying: You might as well argue that

All rabbits are very fast runners.  
Some horses are very fast runners.  
 Therefore some horses are rabbits.

We might continue: You cannot seriously defend this argument, because here there is no question about the facts. The premises are known to be true and the conclusion is known to be false. Your argument is of the same

<sup>\*</sup>We assume, as noted above, that the constituent propositions are themselves contingent, that is, neither logically true (e.g., "All easy chairs are chairs") nor logically false (e.g., "Some easy chairs are not chairs"). The reason for the assumption is this: If it contained either a logically false premise or a logically true conclusion, then the argument would be valid regardless of its syllogistic form—valid in that it would be logically impossible for its premises to be true and its conclusion false. We also assume that the only logical relations among the terms of the syllogism are those asserted or entailed by its premises. The point of these restrictions is to limit our considerations in this chapter and the next to syllogistic arguments alone and to exclude other kinds of arguments whose validity turns on more complex logical considerations that are not appropriate to introduce at this place.



Any standard-form syllogism is completely described when we specify its mood and its figure. The syllogism we have been using as an example is in the second figure; “cowards,” the middle term, is the predicate term of both premises. Its mood, as we pointed out, is **EIO**. So it is completely described as being a syllogism of the form **EIO-2**. It is a valid syllogism, as we noted; every valid syllogistic form, as we shall see, has its own name. The name of this form, **EIO-2**, is *Festino*. We say of this syllogism that it is “in *Festino*.”

Here is another example:

No *M* is *P*  
 All *S* is *M*  
 ∴ No *S* is *P*

This syllogism is in the first figure (its middle term is the subject of the major premise and the predicate of the minor premise); its mood is **EAE**. So we may characterize it completely as **EAE-1**, a form whose unique name is *Celarent*. Any syllogism of this form is “in *Celarent*,” just as any syllogism of the earlier form is “in *Festino*.” And because *Celarent* (**EAE-1**) and *Festino* (**EIO-2**) are known to be *valid* forms, we may conclude that whenever we encounter an argument in one of these forms, it too is valid.

With these analytical tools we can identify every possible categorical syllogism by mood and figure. If we were to list all the possible moods, beginning with **AAA**, **AAE**, **AAI**, **AAO**, **AEA**, **AEE**, . . . , and so on, continuing until every possibility had been named, we would eventually (upon reaching **OOO**) have enumerated sixty-four possible moods. Each mood can occur in each of the four figures;  $4 \times 64 = 256$ . It is certain, therefore, that there are exactly 256 distinct forms that standard-form syllogisms may assume.

Of these 256 possible forms, as we shall see, only a few are valid forms. And each of those valid forms has a unique name, as will be explained.

### EXERCISES

Rewrite each of the following syllogisms in standard form, and name its mood and figure. (*Procedure*: first, identify the conclusion; second, note its predicate term, which is the major term of the syllogism; third, identify the major premise, which is the premise containing the major term; fourth, verify that the other premise is the minor premise by checking to see that it contains the minor term, which is the subject term of the conclusion; fifth, rewrite the argument in standard form—major premise first,

minor premise second, conclusion last; sixth, name the mood and figure of the syllogism.)

### EXAMPLE

1. No nuclear-powered submarines are commercial vessels, so no warships are commercial vessels, because all nuclear-powered submarines are warships.

### SOLUTION

- Step 1. The conclusion is “No warships are commercial vessels.”
  - Step 2. “Commercial vessels” is the predicate term of this conclusion and is therefore the major term of the syllogism.
  - Step 3. The major premise, the premise that contains this term, is “No nuclear-powered submarines are commercial vessels.”
  - Step 4. The remaining premise, “All nuclear-powered submarines are warships,” is indeed the minor premise, because it does contain the subject term of the conclusion, “warships.”
  - Step 5. In standard form this syllogism is written thus:  
 No nuclear-powered submarines are commercial vessels.  
 All nuclear-powered submarines are warships.  
 Therefore no warships are commercial vessels.
  - Step 6. The three propositions in this syllogism are, in order, **E**, **A**, and **E**. The middle term, “nuclear-powered submarines,” is the subject term of both premises, so the syllogism is in the *third* figure. The mood and figure of the syllogism therefore are **EAE-3**.
2. Some evergreens are objects of worship, because all fir trees are evergreens, and some objects of worship are fir trees.
  3. All artificial satellites are important scientific achievements; therefore some important scientific achievements are not U.S. inventions, inasmuch as some artificial satellites are not U.S. inventions.
  4. No television stars are certified public accountants, but all certified public accountants are people of good business sense; it follows that no television stars are people of good business sense.
  - \*5. Some conservatives are not advocates of high tariff rates, because all advocates of high tariff rates are Republicans, and some Republicans are not conservatives.

## OVERVIEW

### The Parts of a Standard-Form Categorical Syllogism

Major Term	The predicate term of the conclusion.
Minor Term	The subject term of the conclusion.
Middle Term	The term that appears in both premises but not in the conclusion.
Major Premise	The premise containing the major term.
Minor Premise	The premise containing the minor term.

A syllogism is in standard form, we said, when its premises are arranged in a specified standard order. Now we can state that order: *In a standard-form syllogism, the major premise is always stated first, the minor premise second, and the conclusion last.* The reason for the importance of this order will soon become clear.

### B. THE MOOD OF THE SYLLOGISM

Every syllogism has a mood. The mood of a syllogism is determined by the types (A, E, I, or O) of standard-form categorical propositions it contains. The mood of the syllogism is therefore represented by three letters, and those three letters are always given in standard-form order. That is, the first letter names the type of the syllogism's major premise; the second letter names the type of the syllogism's minor premise; the third letter names the type of the syllogism's conclusion. In our example syllogism, the major premise ("No heroes are cowards") is an E proposition; the minor premise ("Some soldiers are cowards") is an I proposition; the conclusion ("Some soldiers are not heroes") is an O proposition. Therefore the mood of this syllogism is EIO.

### C. THE FIGURE OF THE SYLLOGISM

The mood of a standard-form syllogism is not enough, by itself, to characterize its logical form. This can be shown by comparing two syllogisms, A and B, with the same mood, which are logically very different.

<p>A.</p> <table> <tr> <td>Major Term</td> <td>Middle Term</td> </tr> <tr> <td>All great scientists are college graduates.</td> <td></td> </tr> <tr> <td>Minor Term</td> <td>Middle Term</td> </tr> <tr> <td>Some professional athletes are college graduates.</td> <td></td> </tr> <tr> <td>Minor Term</td> <td>Major Term</td> </tr> <tr> <td>Therefore some professional athletes are great scientists.</td> <td></td> </tr> </table>	Major Term	Middle Term	All great scientists are college graduates.		Minor Term	Middle Term	Some professional athletes are college graduates.		Minor Term	Major Term	Therefore some professional athletes are great scientists.		<p>B.</p> <table> <tr> <td>Middle Term</td> <td>Major Term</td> </tr> <tr> <td>All artists are egotists.</td> <td></td> </tr> <tr> <td>Middle Term</td> <td>Minor Term</td> </tr> <tr> <td>Some artists are paupers.</td> <td></td> </tr> <tr> <td>Minor Term</td> <td>Major Term</td> </tr> <tr> <td>Therefore some paupers are egotists.</td> <td></td> </tr> </table>	Middle Term	Major Term	All artists are egotists.		Middle Term	Minor Term	Some artists are paupers.		Minor Term	Major Term	Therefore some paupers are egotists.	
Major Term	Middle Term																								
All great scientists are college graduates.																									
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Minor Term	Major Term																								
Therefore some professional athletes are great scientists.																									
Middle Term	Major Term																								
All artists are egotists.																									
Middle Term	Minor Term																								
Some artists are paupers.																									
Minor Term	Major Term																								
Therefore some paupers are egotists.																									

Both of these are of mood AII, but one of them is valid and the other is not. The difference in their forms can be shown most clearly if we display their logical "skeletons" by abbreviating the minor terms as *S* (subject of the conclusion), the major terms as *P* (predicate of the conclusion), and the middle terms as *M*. Using the three-dot symbol "∴" for "therefore," we get these skeletons:

<p>A. All <i>P</i> is <i>M</i>. Some <i>S</i> is <i>M</i>. ∴ Some <i>S</i> is <i>P</i>.</p>	<p>B. All <i>M</i> is <i>P</i>. Some <i>M</i> is <i>S</i>. ∴ Some <i>S</i> is <i>P</i>.</p>
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These are very different. In the one labeled A, the middle term, *M*, is the predicate term of both premises; but in the one labeled B, the middle term, *M*, is the subject term of both premises. Syllogism B will be seen to be a valid argument; syllogism A, on the other hand, is invalid.

These examples show that although the form of a syllogism is partially described by its mood (AII in both of these cases), syllogisms that have the same mood may differ importantly in their forms, depending on the relative positions of their middle terms. To describe the form of a syllogism completely we must state its *mood* (the three letters of its three propositions) and its *figure*—where by *figure* we mean the position of the middle term in its premises.

Syllogisms can have four—and only four—possible different figures:

1. The middle term may be the subject term of the major premise and the predicate term of the minor premise; or
2. The middle term may be the predicate term of both premises; or
3. The middle term may be the subject term of both premises; or
4. The middle term may be the predicate term of the major premise and the subject term of the minor premise.

These different possible positions of the middle term constitute the first, second, third, and fourth figures, respectively. Every syllogism must have one or another of these four figures. The characters of these figures may be visualized more readily when the figures are schematized as in the following array, in which reference to mood is suppressed and the quantifiers and copulas are not shown—but the relative positions of the terms of the syllogism are brought out:

<p><i>M</i> — <i>P</i> <i>S</i> — <i>M</i> ∴ <i>S</i> — <i>P</i></p> <p>First Figure</p>	<p><i>P</i> — <i>M</i> <i>S</i> — <i>M</i> ∴ <i>S</i> — <i>P</i></p> <p>Second Figure</p>	<p><i>M</i> — <i>P</i> <i>M</i> — <i>S</i> ∴ <i>S</i> — <i>P</i></p> <p>Third Figure</p>	<p><i>P</i> — <i>M</i> <i>M</i> — <i>S</i> ∴ <i>S</i> — <i>P</i></p> <p>Fourth Figure</p>
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# Categorical Syllogisms

- 6.1 Standard-Form Categorical Syllogisms
- 6.2 The Formal Nature of Syllogistic Argument
- 6.3 Venn Diagram Technique for Testing Syllogisms
- 6.4 Syllogistic Rules and Syllogistic Fallacies
- 6.5 Exposition of the Fifteen Valid Forms of the Categorical Syllogism
- Appendix: Deduction of the Fifteen Valid Forms of the Categorical Syllogism

## 6.1 Standard-Form Categorical Syllogisms

We are now in a position to use categorical propositions in more extended reasoning. Arguments that rely on A, E, I, and O propositions commonly have two categorical propositions as premises and one categorical proposition as a conclusion. Such arguments are called *syllogisms*; a *syllogism* is, in general, a deductive argument in which a conclusion is inferred from two premises.

The syllogisms with which we are concerned here are called *categorical* because they are arguments based on the relations of classes, or categories—relations that are expressed by the categorical propositions with which we are familiar. More formally, we define a *categorical syllogism* as a deductive argument consisting of three categorical propositions that together contain exactly three terms, each of which occurs in exactly two of the constituent propositions.

Syllogisms are very common, very clear, and readily testable. The system of categorical syllogisms that we will explore is powerful and deep. The seventeenth-century philosopher and mathematician Gottfried Leibniz said, of the invention of the form of syllogisms, that it was “one of the most beautiful and also one of the most important made by the human mind.” Syllogisms are the workhorse arguments with which deductive logic, as traditionally practiced, has been made effective in writing and in controversy.

It will be convenient to have an example to use as we discuss the parts and features of the syllogism. Here is a valid standard-form categorical syllogism that we shall use as illustrative:

No heroes are cowards.  
Some soldiers are cowards.  
Therefore some soldiers are not heroes.

To analyze such an argument accurately, it needs to be in *standard form*. A *categorical syllogism* is said to be in standard form (as the above sample is) when two things are true of it: (1) its premises and its conclusion are all standard-form categorical propositions (A, E, I, or O); and (2) those propositions are arranged in a specified *standard order*. The importance of this standard form will become evident when we turn to the task of testing the validity of syllogisms.

To explain the order of the premises that is required to put any syllogism into standard form, we need the *logical names* of the *premises* of the syllogism, and the names of the *terms* of the syllogism, and we must understand why those names—very useful and very important—are assigned to them. This is the next essential step in our analysis of categorical syllogisms.\*

### A. TERMS OF THE SYLLOGISM: MAJOR, MINOR, AND MIDDLE

The three categorical propositions in our example argument above contain exactly three terms: *heroes*, *soldiers*, and *cowards*. To identify the terms by name, we look to the conclusion of the syllogism, which of course contains exactly two terms. The conclusion in our sample is an O proposition, “Some soldiers are not heroes.” The term that occurs as the *predicate* of the conclusion (“heroes,” in this case) is called the **major term** of the syllogism. The term that occurs as the *subject* of the conclusion (“soldiers” in this case) is called the **minor term** of the syllogism. The third term of the syllogism (“cowards” in this case), which never occurs in the conclusion but always appears in both premises, is called the **middle term**.

The premises of a syllogism also have names. Each premise is named after the term that appears in it. The major term and the minor term must each occur in a different premise. The premise containing the major term is called the **major premise**. In the example, “heroes” is the major term, so the premise containing “heroes”—“No heroes are cowards”—is the major premise. It is the major premise not because it appears first, but only because it is the premise that contains the major term; it would be the major premise no matter in what order the premises were written.

The premise containing the minor term is called the **minor premise**. In the example, “soldiers” is the minor term, so the premise containing “soldiers”—“Some soldiers are cowards”—is the minor premise. It is the minor premise not because of its position, but because it is the premise that contains the minor term.

\*In this chapter, for the sake of brevity we will refer to categorical syllogisms simply as “syllogisms,” even though there are other kinds of syllogisms that will be discussed in later chapters.

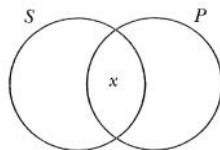


## ■ EXAMPLE

1. Some sculptors are painters.

## ■ SOLUTION

$SP \neq 0$



2. No peddlers are millionaires.
3. All merchants are speculators.
4. Some musicians are not pianists.
- \*5. No shopkeepers are members.
6. Some political leaders of high reputation are scoundrels.
7. All physicians licensed to practice in this state are medical school graduates who have passed special qualifying examinations.
8. Some stockbrokers who advise their customers about making investments are not partners in companies whose securities they recommend.
9. All puritans who reject all useless pleasure are strangers to much that makes life worth living.
- \*10. No modern paintings are photographic likenesses of their objects.
11. Some student activists are middle-aged men and women striving to recapture their lost youth.
12. All medieval scholars were pious monks living in monasteries.
13. Some state employees are not public-spirited citizens.
14. No magistrates subject to election and recall will be punitive tyrants.
- \*15. Some patients exhibiting all the symptoms of schizophrenia have bipolar disorder.
16. Some passengers on large jet airplanes are not satisfied customers.
17. Some priests are militant advocates of radical social change.
18. Some stalwart defenders of the existing order are not members of a political party.
19. No pipelines laid across foreign territories are safe investments.
- \*20. All pornographic films are menaces to civilization and decency.

## SUMMARY

This chapter has introduced and explained the basic elements of classical, or Aristotelian, deductive logic, as distinguished from modern symbolic logic. (See Section 5.1.)

In Section 5.2 we introduced the concept of classes, on which traditional logic is built, and the categorical propositions that express relations between classes.

In Section 5.3 we explained the four basic standard-form categorical propositions:

- A: universal affirmative
- E: universal negative
- I: particular affirmative
- O: particular negative

In Section 5.4 we discussed various features of these standard-form categorical propositions: their quality (affirmative or negative) and their quantity (universal or particular). We also explained why different terms are distributed or undistributed, in each of the four basic kinds of propositions.

In Section 5.5 we explored the kinds of opposition arising among the several standard-form categorical propositions: what it means for propositions to be contradictories, or contraries, or subcontraries, or sub- and superalterns of one another. We showed how these relations are exhibited on the traditional square of opposition, and explained the immediate inferences that can be drawn from them.

In Section 5.6 we examined other kinds of immediate inferences that are based on categorical propositions: conversion, obversion, and contraposition.

In Section 5.7 we explored the controversial issue of existential import, showing that the traditional square of opposition can be retained only if we make a blanket assumption that the classes to which the subjects of propositions refer always do have some members—an assumption that modern logicians are unwilling to make. We then explained the interpretation of propositions to be adopted throughout this book, called Boolean, which retains much, but not all, of the traditional square of opposition while rejecting the blanket assumption of nonempty classes. In this Boolean interpretation, we explained that particular propositions (I and O propositions) are interpreted as having existential import, whereas universal propositions (A and E propositions) are interpreted as not having such import. We carefully detailed the consequences of adopting this interpretation of propositions.

In Section 5.8 we returned to the use of Venn diagrams, using intersecting circles to represent classes. We showed how, with additional markings, Venn diagrams may also be used to represent categorical propositions.

This chapter has provided the tools we will need to analyze categorical syllogisms, of which standard-form propositions are the essential building blocks.

Everest. All these classes are diagrammed in Figure 5-6, where the letters  $S$  and  $P$  are interpreted as in this paragraph.

Diagrams of this kind, as noted earlier, are called *Venn diagrams* after John Venn, the English logician who introduced this notation. When, in such diagrams, the several areas are labeled, but not marked in any other way, they represent *classes* only. Figure 5-6 illustrates this. It does not represent any proposition. In such a diagram, if a circle or part of a circle is blank, that signifies nothing—neither that there are, nor that there are not, members of the class represented by that space.

With certain additions, however, Venn diagrams can be used to represent *propositions* as well as classes. By shading out some spaces, or by inserting  $x$ 's in various parts of the picture, we can accurately diagram any one of the four standard-form categorical propositions. Because Venn diagrams (with appropriate markings) represent categorical propositions so fully and so graphically, these diagrams have become one of the most powerful and most widely used instruments for the appraisal of syllogistic arguments. Let us consider how each of the four basic categorical propositions can be represented using this technique.

To diagram the **A** proposition, "All  $S$  is  $P$ ," symbolized by  $S\bar{P} = 0$ , we simply shade out the part of the diagram that represents the class  $S\bar{P}$ , thus indicating that it has no members or is empty. To diagram the **E** proposition, "No  $S$  is  $P$ ," symbolized by  $SP = 0$ , we shade out the part of the diagram that represents the class  $SP$ , to indicate that it is empty. To diagram the **I** proposition, "Some  $S$  is  $P$ ," symbolized by  $SP \neq 0$ , we insert an  $x$  into the part of the diagram that represents the class  $SP$ . This insertion indicates that the class product is not empty but has at least one member. Finally, for the **O** proposition, "Some  $S$  is not  $P$ ," symbolized by  $S\bar{P} \neq 0$ , we insert an  $x$  into the part of the diagram that represents the class  $S\bar{P}$ , to indicate that it is not empty but has at least one member. Placed side by side, diagrams for the four standard-form categorical propositions display their different meanings very clearly, as shown in Figure 5-7.

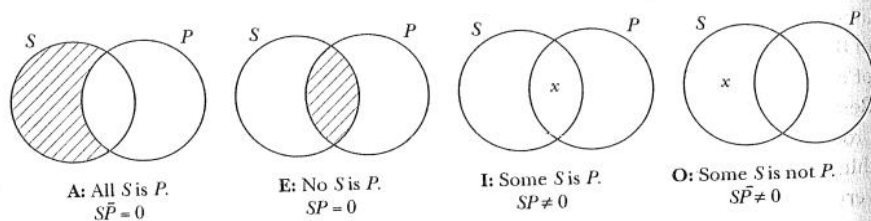


Figure 5-7

We have constructed diagrammatic representations for "No  $S$  is  $P$ " and "Some  $S$  is  $P$ ," and because these are logically equivalent to their converses, "No  $P$  is  $S$ " and "Some  $P$  is  $S$ ," the diagrams for the latter have already been shown. To diagram the **A** proposition, "All  $P$  is  $S$ ," symbolized by  $P\bar{S} = 0$  within the same framework, we must shade out the part of the diagram that represents the class  $P\bar{S}$ . It should be obvious that the class  $P\bar{S}$  is the same as the class  $\bar{S}P$ —if not immediately, then by recognizing that every object that belongs to the class of all painters and the class of all non-Spaniards must (also) belong to the class of all non-Spaniards and the class of all painters—all painting non-Spaniards are non-Spanish painters, and vice versa. And to diagram the **O** proposition, "Some  $P$  is not  $S$ ," symbolized by  $P\bar{S} \neq 0$ , we insert an  $x$  into the part of the diagram that represents the class  $P\bar{S}$  ( $= \bar{S}P$ ). Diagrams for these propositions then appear as shown in Figure 5-8.

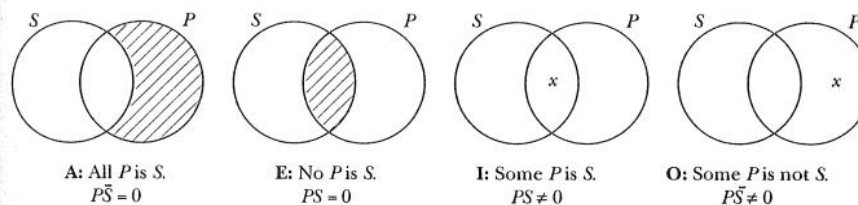


Figure 5-8

This further adequacy of the two-circle diagrams is mentioned because in the next chapter it will be important to be able to use a given pair of overlapping circles with given labels—say,  $S$  and  $M$ —to diagram any standard-form categorical proposition containing  $S$  and  $M$  as its terms, regardless of the order in which they occur in it.

The Venn diagrams constitute an *iconic* representation of the standard-form categorical propositions, in which spatial inclusions and exclusions correspond to the nonspatial inclusions and exclusions of classes. They provide an exceptionally clear method of notation. They also provide the basis for the simplest and most direct method of testing the validity of categorical syllogisms, as will be explained in Chapter 6.

### EXERCISES

Express each of the following propositions as equalities or inequalities, representing each class by the first letter of the English term designating it, and symbolizing the proposition by means of a Venn diagram.

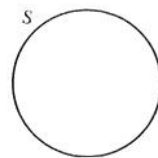


Figure 5-3

The diagram in Figure 5-3 is of a class, not a proposition. It represents the class  $S$ , but it says nothing about it. To diagram the proposition that  $S$  has no members, or that there are no  $S$ 's, we shade all of the interior of the circle representing  $S$ , indicating in this way that it contains nothing and is empty. To diagram the proposition that there are  $S$ 's, which we interpret as saying that there is at least one member of  $S$ , we place an  $x$  anywhere in the interior of the circle representing  $S$ , indicating in this way that there is something inside it, that it is not empty. Thus the two propositions, "There are no  $S$ 's," and "There are  $S$ 's," are represented by the two diagrams in Figure 5-4.

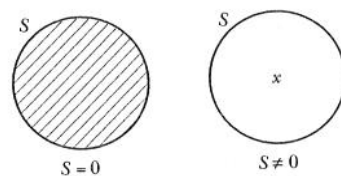


Figure 5-4

Note that the circle that diagrams the class  $S$  will also, in effect, diagram the class  $\bar{S}$ , for just as the interior of the circle represents all members of  $S$ , so the exterior of the circle represents all members of  $\bar{S}$ .

To diagram a standard-form categorical proposition, as explained in Section 5.3, two circles are required. Figure 5-5 shows a pair of intersecting circles, which we may use as the skeleton, or framework, for diagramming any standard-form categorical proposition whose subject terms are abbreviated by  $S$  and  $P$ .

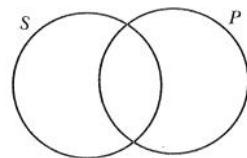


Figure 5-5

Figure 5-5 diagrams the two classes of  $S$  and  $P$  but diagrams no proposition concerning them. It does not affirm that either or both have members, nor does it deny that they have. As a matter of fact, there are more than two classes diagrammed by the two intersecting circles. The part of the circle labeled  $S$  that does not overlap the circle labeled  $P$  diagrams all  $S$ 's that are not  $P$ 's and can be thought of as representing the product of the classes  $S$  and  $\bar{P}$ . We may label it  $S\bar{P}$ . The overlapping part of the two circles represents the product of the classes  $S$  and  $P$ , and diagrams all things belonging to both of them. It is labeled  $SP$ . The part of the circle labeled  $P$  that does not overlap the circle labeled  $S$  diagrams all  $P$ 's that are not  $S$ 's, and represents the product of the class  $\bar{S}$  and  $P$ . It is labeled  $\bar{S}P$ . Finally, the part of the diagram external to both circles represents all things that are neither in  $S$  nor in  $P$ ; it diagrams the fourth class  $\bar{S}\bar{P}$ , so labeled. With these labels inserted, Figure 5-5 becomes Figure 5-6.

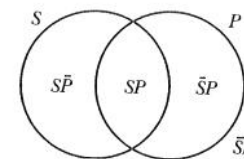


Figure 5-6

Figure 5-6 can be interpreted in terms of the several different classes determined by the class of all Spaniards ( $S$ ) and the class of all painters ( $P$ ).  $SP$  is the product of these two classes, containing all those things and only those things that belong to both of them. Every member of  $SP$  must be a member of both  $S$  and  $P$ ; every member must be both a Spaniard and a painter. This product class  $SP$  is the class of all Spanish painters, which contains, among others, Velázquez and Goya.  $S\bar{P}$  is the product of the first class and the complement of the second, containing all those things and only those things that belong to the class  $S$  but not to the class  $P$ . It is the class of all Spaniards who are not painters, all Spanish nonpainters, and it will contain neither Velázquez nor Goya, but it will include both the novelist Cervantes and the dictator Franco, among many others.  $\bar{S}P$  is the product of the second class and the complement of the first, and is the class of all painters who are not Spaniards. This class  $\bar{S}P$  of all non-Spanish painters includes, among others, both the Dutch painter Rembrandt and the American painter Georgia O'Keeffe. Finally,  $\bar{S}\bar{P}$  is the product of the complements of the two original classes. It contains all those things and only those things that are neither Spaniards nor painters. It is a very large class indeed, containing not merely English admirals and Swiss mountain climbers, but such things as the Mississippi River and Mount



letter *P* designates the class of all poems, then the class of all things that are both satires and poems is represented by the symbol *SP*, which thus designates the class of all satirical poems (or poetic satires). The common part or common membership of two classes is called the *product* or *intersection* of the two classes. The *product* of two classes is the class of all things that belong to both of them. The product of the class of all Americans and the class of all composers is the class of all American composers. (One must be on one's guard against certain oddities of the English language here. For example, the product of the class of all Spaniards and the class of all dancers is not the class of all Spanish dancers, for a Spanish dancer is not a dancer who is Spanish, but any person who performs Spanish dances. Similarly, with abstract painters, English majors, antique dealers, and so on.)

This new notation permits us to symbolize *E* and *I* propositions as equations and inequalities. The *E* proposition, "No *S* is *P*," says that no members of the class *S* are members of the class *P*; that is, there are no things that belong to both classes. This can be rephrased by saying that the product of the two classes is empty, which is symbolized by the equation  $SP = 0$ . The *I* proposition, "Some *S* is *P*," says that at least one member of *S* is also a member of *P*. This means that the product of the classes *S* and *P* is not empty and is symbolized by the inequality  $SP \neq 0$ .

To symbolize *A* and *O* propositions, it is convenient to introduce a new method of representing class complements. The complement of a class is the collection or class of all things that do not belong to the original class, as explained in Section 5.6. The complement of the class of all soldiers is the class of all things that are not soldiers—the class of all nonsoldiers. Where the letter *S* symbolizes the class of all soldiers, we symbolize the class of all nonsoldiers by  $\bar{S}$  (read "S bar"), the symbol for the original class with a bar above it. The *A* proposition, "All *S* is *P*," says that all members of the class *S* are also members of the class *P*; that is, that there are no members of the class *S* that are not members of *P* or (by obversion) that "No *S* is non-*P*." This, like any other *E* proposition, says that the product of the classes designated by its subject and predicate terms is empty. It is symbolized by the equation  $S\bar{P} = 0$ . The *O* proposition, "Some *S* is not *P*," obverts to the logically equivalent *I* proposition, "Some *S* is non-*P*," which is symbolized by the inequality  $S\bar{P} \neq 0$ .

In their symbolic formulations, the interrelations among the four standard-form categorical propositions appear very clearly. It is obvious that the *A* and *O* propositions are contradictories when they are symbolized as  $S\bar{P} = 0$  and  $S\bar{P} \neq 0$ , and it is equally obvious that the *E* and *I* propositions,  $SP = 0$  and  $SP \neq 0$ , are contradictories. The *Boolean square of opposition* may be represented as shown in Figure 5-2.

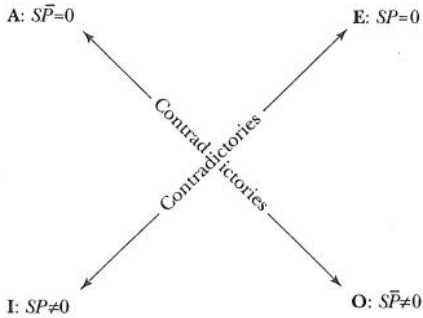


Figure 5-2 The Boolean Square of Opposition

Symbolic Representation of Categorical Propositions			
Form	Proposition	Symbolic Representation	Explanation
A	All <i>S</i> is <i>P</i> .	$S\bar{P} = 0$	The class of things that are both <i>S</i> and non- <i>P</i> is empty.
E	No <i>S</i> is <i>P</i> .	$SP = 0$	The class of things that are both <i>S</i> and <i>P</i> is empty.
I	Some <i>S</i> is <i>P</i> .	$SP \neq 0$	The class of things that are both <i>S</i> and <i>P</i> is not empty. ( <i>SP</i> has at least one member.)
O	Some <i>S</i> is not <i>P</i> .	$S\bar{P} \neq 0$	The class of things that are both <i>S</i> and non- <i>P</i> is not empty. ( $S\bar{P}$ has at least one member.)

The notation shown in the table is useful, for example, in representing the relationship among contradictories in the Boolean square of opposition.

When first explaining the four types of standard-form categorical propositions, in Section 5.3, we represented the relations of the classes in those propositions graphically with intersecting circles, labeled *S* and *P*. Now we carry that process of diagramming categorical propositions somewhat further, enriching our notation in ways that will facilitate the analysis to follow. We begin by representing any class with an unmarked circle, labeled with the term that designates that class. The class *S* is diagrammed with a simple circle, as shown in Figure 5-3.

## EXERCISES

In the preceding discussion of existential import, it was shown why, in the Boolean interpretation of propositions adopted in this book, some inferences, which traditionally were thought to be valid, mistakenly assume that certain classes have members; these inferences commit the existential fallacy and are not valid. This existential fallacy is committed in each of the following arguments; explain the point at which, in each argument, the mistaken existential assumption is made.

## EXAMPLE

- A. (1) No mathematician is one who has squared the circle.  
 therefore, (2) No one who has squared the circle is a mathematician;  
 therefore, (3) All who have squared the circle are nonmathematicians;  
 therefore, (4) Some nonmathematician is one who has squared the circle.

## SOLUTION

Step (3) to step (4) is invalid. The inference at this point is conversion by limitation (that is, from *All S is P* to *Some P is S*), which was acceptable in the traditional interpretation but is invalid in the Boolean interpretation. This step relies on an inference from a universal proposition to a particular proposition, but the preceding discussion has shown that the classes in a universal proposition cannot be assumed to have members, whereas the classes in a particular proposition do have members. Thus the invalid passage from (3) to (4) permits the inference that the predicate class in (4) is not empty, and therefore that there *is* someone who has squared the circle! In inferring (4) from (3), one commits the existential fallacy.

- B. (1) No citizen is one who has succeeded in accomplishing the impossible;  
 therefore, (2) No one who has succeeded in accomplishing the impossible is a citizen;  
 therefore, (3) All who have succeeded in accomplishing the impossible are noncitizens;  
 therefore, (4) Some who have succeeded in accomplishing the impossible are noncitizens;  
 therefore, (5) Some noncitizen is one who has succeeded in accomplishing the impossible.

- C. (1) No acrobat is one who can lift himself by his own bootstraps;  
 therefore, (2) No one who can lift himself by his own bootstraps is an acrobat;  
 therefore, (3) Someone who can lift himself by his own bootstraps is not an acrobat. (From which it follows that there is at least one being who can lift himself by his own bootstraps.)
- D. (1) It is true that: No unicorns are animals found in the Bronx Zoo;  
 therefore, (2) It is false that: All unicorns are animals found in the Bronx Zoo;  
 therefore (3) It is true that: Some unicorns are not animals found in the Bronx Zoo. (From which it follows that there exists at least one unicorn.)
- \*E. (1) It is false that: Some mermaids are members of college sororities;  
 therefore (2) It is true that: Some mermaids are not members of college sororities. (From which it follows that there exists at least one mermaid.)

## 5.8 Symbolism and Diagrams for Categorical Propositions

Because the Boolean interpretation of categorical propositions depends heavily on the notion of an empty class, it is convenient to have a special symbol to represent it. The zero symbol, 0, is used for this purpose. To say that the class designated by the term *S* has no members, we write an equals sign between *S* and 0. Thus the equation  $S = 0$  says that there are no *S*'s, or that *S* has no members.

To say that the class designated by *S* does have members is to deny that *S* is empty. To assert that there are *S*'s is to deny the proposition symbolized by  $S = 0$ . We symbolize that denial by drawing a slanting line through the equals sign. Thus the inequality  $S \neq 0$  says that there are *S*'s, by denying that *S* is empty.

Standard-form categorical propositions refer to two classes, so the equations that represent them are somewhat more complicated. Where each of two classes is already designated by a symbol, the class of all things that belong to both of them can be represented by juxtaposing the symbols for the two original classes. For example, if the letter *S* designates the class of all satires and the

members in the classes we are talking about. If you say, for example, "All trespassers will be prosecuted," far from presupposing that the class of trespassers has members, you will ordinarily be intending to ensure that the class will become and remain empty!

Third, in science, and in other theoretical spheres, we often wish to reason without making any presuppositions about existence. Newton's first law of motion, for example, asserts that certain things are true about bodies that are not acted on by any external forces: They remain at rest, or they continue their straight-line motion. The law may be true; a physicist may wish to express and defend it without wanting to presuppose that there actually are any bodies that are not acted on by external forces.

Objections of this kind make the blanket existential presupposition unacceptable for modern logicians. The Aristotelian interpretation of categorical propositions, long thought to be correct, must be abandoned, and a more modern interpretation employed.

In modern logic it is not assumed that the classes to which categorical propositions refer always have members. The modern interpretation that explicitly rejects this assumption is called, as we noted earlier, Boolean.\* We adopt the Boolean interpretation of categorical propositions in all that follows. This has important logical consequences. Therefore we set forth now what this Boolean interpretation of categorical propositions entails.

1. In some respects, the traditional interpretation is not upset. *I* and *O* propositions continue to have existential import in the Boolean interpretation, so the proposition "Some *S* is *P*" is false if the class *S* is empty, and the proposition "Some *S* is not-*P*" is likewise false if the class *S* is empty.
2. It also remains true in this interpretation that the universal propositions, *A* and *E*, are the contradictories of the particular propositions, *O* and *I*. That is, the proposition "All men are mortal" does contradict the proposition "Some men are not mortal," and the proposition "No gods are mortal" does contradict the proposition "Some gods are mortal."
3. All this is entirely coherent because, in the Boolean interpretation, universal propositions are interpreted as having no existential import. So even when the *S* class is empty, the proposition "All *S* is *P*" can be true, as can the proposition "No *S* is *P*." For example, the propositions "All unicorns have horns" and "No unicorns have wings" may both be true, even if

\*Bertrand Russell, another of the founders of modern symbolic logic, also advanced this approach in a famous essay entitled "The Existential Import of Propositions," in *Mind*, July 1905, and referred to it there as "Peano's interpretation" of propositions, after Giuseppe Peano, a great Italian mathematician of the early twentieth century.

there are no unicorns. But if there are no unicorns, the *I* proposition, "Some unicorns have horns," is false, as is the *O* proposition, "Some unicorns do not have wings."

4. Sometimes, in ordinary discourse, we utter a universal proposition with which we do intend to assert existence. The Boolean interpretation permits this to be expressed, but doing so requires two propositions, one existential in force but particular, the other universal but not existential in force.
5. Some very important changes result from our adoption of the Boolean interpretation. Corresponding *A* and *E* propositions can both be true and are therefore not contraries. This may seem paradoxical and will be explained in detail later, in Sections 10.2 and 10.3. For the present it will suffice to say that, in the Boolean interpretation, "All unicorns have wings" is taken to assert that "If there is a unicorn, then it has wings," and "No unicorns have wings" is taken to assert that "If there is a unicorn, it does not have wings." And both of these "if. . . then" propositions can be true if indeed there are no unicorns.
6. In like manner, in the Boolean interpretation, corresponding *I* and *O* propositions, because they do have existential import, can both be false if the subject class is empty. So corresponding *I* and *O* propositions are not subcontraries.
7. In the Boolean interpretation, subalternation—inferring an *I* proposition from its corresponding *A*, and an *O* proposition from its corresponding *E*—is not generally valid. This is because, plainly, one may not validly infer a proposition that has existential import from one that does not.
8. The Boolean interpretation preserves some immediate inferences: conversion for *E* and for *I* propositions is preserved; contraposition for *A* and for *O* propositions is preserved; obversion for any proposition is preserved. But conversion by limitation, and contraposition by limitation, are not generally valid.
9. The traditional square of opposition, in the Boolean interpretation, is transformed in the following general way: Relations along the sides of the square are undone, but the diagonal, contradictory relations remain in force.

In short, the blanket existential presupposition is rejected by modern logicians. It is a mistake, we hold, to assume that a class has members if it is not asserted explicitly that it does. Any argument that relies on this mistaken assumption is said to commit the fallacy of existential assumption, or more briefly, the **existential fallacy**. With this Boolean interpretation clearly in mind, we are now in a position to set forth a powerful system for the symbolizing and diagramming of standard-form categorical syllogisms.



O propositions have existential import, and they follow validly from their corresponding A and E propositions, then A and E propositions must *also* have existential import, because a proposition with existential import cannot be derived validly from another that does not have such import.\*

This consequence creates a very serious problem. We know that A and O propositions, on the traditional square of opposition, are contradictories. "All Danes speak English" is contradicted by "Some Danes do not speak English." Contradictories cannot both be true, because one of the pair must be false; nor can they both be false, because one of the pair must be true. But if corresponding A and O propositions do have existential import, as we concluded in the paragraph just above, then both contradictories *could* be false! To illustrate, the A proposition, "All inhabitants of Mars are blond," and its corresponding O proposition, "Some inhabitants of Mars are not blond," are contradictories; if they have existential import—that is, if we interpret them as asserting that there *are* inhabitants of Mars—then both these propositions are false if Mars has no inhabitants. And, of course, we do know that Mars has no inhabitants; the class of its inhabitants is empty, so both of the propositions in the example are false. But if they are both false, they *cannot be contradictories!*

Something seems to have gone wrong with the traditional square of opposition in cases of this kind. If the traditional square is correct when it tells us that A and E propositions validly imply their corresponding I and O propositions, then the square is not correct when it tells us that corresponding A and O propositions are contradictories. And in that case, the square is also mistaken in holding that the corresponding I and O propositions are subcontraries.

What is to be done? Can the traditional square of opposition be rescued? Yes, it can, but the price is high. We could rehabilitate the traditional square of opposition by introducing the notion of a *presupposition*. Much earlier (in Section 4.5), we observed that some complex questions are properly answered "yes" or "no" only if the answer to a prior question has been presupposed. "Did you spend the money you stole?" can be reasonably answered "yes" or "no" only if the presupposition that you stole some money is granted. Now, to rescue the square of opposition, we might insist that *all* propositions—that is, the four standard-form categorical propositions A, E, I, and O—presuppose (in the sense indicated above) that the classes to which they refer do have

\*There is another way to show that the existential import of A and E propositions must follow from that of I and O propositions, on the traditional square of opposition. In the case of the A proposition, we could show it by relying on the (traditionally assumed) validity of conversion by limitation; in the case of the E proposition, we could show it by relying on the (traditionally assumed) validity of contraposition by limitation. The result is always the same as that reached above: On the traditional square of opposition, if I and O propositions have existential import, A and E propositions must also have existential import.

members; they are not empty. That is, questions about the truth or falsehood of propositions, and about the logical relations holding among them, are admissible and may be reasonably answered (in this interpretation) only if we presuppose that they never refer to empty classes. In this way, we may save all of the relationships set forth in the traditional square of opposition: A and E will remain contraries, I and O will remain subcontraries, subalterns will follow validly from their superalterns, and A and O will remain contradictories, as will I and E. To achieve this result, however, we must pay by making the blanket presupposition that all classes designated by our terms (and the complements of these classes) do have members—are not empty.\*

Well, why not do just that? This existential presupposition is both necessary and sufficient to rescue Aristotelian logic. It is, moreover, a presupposition in full accord with the ordinary use of modern languages such as English in very many cases. If you are told, "All the apples in the barrel are Delicious," and you find when you look into the barrel that it is empty, what can you say? You would probably not say that the claim is false, or true, but would instead point out that there *are* no apples in the barrel. You would thus be explaining that the speaker had made a mistake, that in this case the existential presupposition (that there exist apples in the barrel) was false. And the fact that we would respond in this corrective fashion shows that we do understand, and do generally accept, the existential presupposition of propositions that are ordinarily uttered.

Unfortunately, this blanket existential presupposition, introduced to rescue the traditional square of opposition, imposes intellectual penalties that are too heavy to bear. There are very good reasons *not* to do it. Here are three such reasons.

First, this rescue preserves the traditional relations among A, E, I, and O propositions, but only at the cost of reducing their power to formulate assertions that we may need to formulate. If we invariably presuppose that the class designated has members, *we will never be able to formulate the proposition that denies that it has members!* And such denials may sometimes be very important and must surely be made intelligible.

Second, even ordinary usage of language is not in complete accord with this blanket presupposition. *Sometimes what we say does not suppose that there are*

\*Phillip H. Wiebe argues that Aristotelian logic does not require the assumption that the class designated by the complement of the subject term be nonempty. See "Existential Assumptions for Aristotelian Logic," *Journal of Philosophical Research* 16 (1990–1991): 321–328. But Aristotelian logic certainly does require the assumption that at least the classes designated by the other three terms (the subject term, the predicate term, and the complement of the predicate term) are not empty—and this existential assumption gives rise to all the difficulties noted in the remarks that follow.

been a source of controversy for literally thousands of years. In this section we explain this problem, and we provide a resolution on which a coherent analysis of syllogisms may be developed.

The issues here, as we shall see, are far from simple, but the analysis of syllogisms in succeeding chapters does not require that the complications of this controversy be mastered. It does require that the interpretation of categorical propositions that emerges from the resolution of the controversy be understood. This is commonly called the **Boolean interpretation** of categorical propositions—named after George Boole (1815–1864), an English mathematician whose contributions to logical theory played a key role in the later development of the modern computer. So if the outcome of the following discussion—summarized in the final two paragraphs of this section on pages 210–211 is fully grasped, the intervening pages of this section may be safely bypassed.

To understand the problem, and the Boolean outcome with which we emerge, it must be seen that some propositions have existential import, and some do not. A proposition is said to have **existential import** if it typically is uttered to assert the existence of objects of some kind. Why should this seemingly abstruse matter be of concern to the student of logic? Because the correctness of the reasoning in many arguments is directly affected by whether the propositions of which those arguments are built do, or do not, have existential import. We must arrive at a clear and consistent interpretation of categorical propositions in order to determine with confidence what may be rightly inferred from them, and to guard against incorrect inferences that are sometimes drawn from them.

We begin with I and O propositions, which surely do have existential import. Thus the I proposition, “Some soldiers are heroes,” says that there exists at least one soldier who is a hero. And the O proposition, “Some dogs are not companions,” says that there exists at least one dog that is not a companion. Particular propositions, I and O propositions, plainly *do* assert that the classes designated by their subject terms (for example, soldiers and dogs) are not empty—the class of soldiers, and the class of dogs (if the examples given here are true), each has at least one member.\*

\*A few propositions appear to be exceptions. “Some ghosts appear in Shakespeare’s plays” and “Some Greek gods are described in the *Iliad*” are particular propositions that are certainly true even though there are neither ghosts nor Greek gods. However, it is the formulation that misleads in such cases. These statements do not themselves affirm the existence of ghosts or Greek gods; they say only that there are certain other propositions that are affirmed or implied in Shakespeare’s plays and in the *Iliad*. The propositions of Shakespeare and Homer may not be true, but it is certainly true that their writings contain or imply those propositions. And that is all that is affirmed by these apparent exceptions, which arise chiefly in literary or mythological contexts. I and O propositions do have existential import.

## VISUAL LOGIC

### Aristotle v. Boole on Interpreting Categorical Propositions



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There are two rival interpretations of categorical propositions: the Aristotelian, which is traditional, and the Boolean, which is modern.

In the interpretation of the ancient Greek philosopher Aristotle, the truth of a universal proposition (“All leprechauns wear little green hats,” or “No frogs are poisonous”) implies the truth of its corresponding particular proposition (“Some leprechauns wear little green hats” or “Some frogs are not poisonous”).

In contrast, George Boole, a nineteenth-century English mathematician, argued that we cannot infer the truth of the particular proposition from the truth of its corresponding universal proposition, because (as both sides agree) every particular proposition asserts the existence of its subject class; if some frogs are not poisonous, there must be at least one frog. But if the universal proposition permits us to infer the corresponding particular proposition, then “All leprechauns wear little green hats” would permit us to infer that some leprechauns do, and that would imply that there really are leprechauns!

So, in the modern or Boolean interpretation, a universal proposition (an A or an E proposition) must be understood to assert only that “If there is such a thing as a leprechaun, it wears a little green hat,” and “If there is such a thing as a frog, it is not poisonous.”



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If this is so, however, if I and O propositions have existential import (as no one would wish to deny), wherein lies the problem? The problem arises from the *consequences* of this fact, which are very awkward. Earlier we saw that an I proposition follows validly from its corresponding A proposition by subalternation. That is, from “All spiders are eight-legged animals,” we infer validly that some spiders are eight-legged animals. And similarly, we said that an O proposition follows validly from its corresponding E proposition. But if I and

known to be true, which can be known to be false, and which are undetermined?

- \*1. Some nonpacifists are not nonsocialists.
- 2. No socialists are nonpacifists.
- 3. All nonsocialists are nonpacifists.
- 4. No nonpacifists are socialists.
- \*5. No nonsocialists are nonpacifists.
- 6. All nonpacifists are nonsocialists.
- 7. No pacifists are nonsocialists.
- 8. Some socialists are not pacifists.
- 9. All pacifists are socialists.
- \*10. Some nonpacifists are socialists.

E. If "No scientists are philosophers" is true, what may be inferred about the truth or falsehood of the following propositions? That is, which can be known to be true, which can be known to be false, and which are undetermined?

- \*1. No nonphilosophers are scientists.
- 2. Some nonphilosophers are not nonscientists.
- 3. All nonscientists are nonphilosophers.
- 4. No scientists are nonphilosophers.
- \*5. No nonscientists are nonphilosophers.
- 6. All philosophers are scientists.
- 7. Some nonphilosophers are scientists.
- 8. All nonphilosophers are nonscientists.
- 9. Some scientists are not philosophers.
- \*10. No philosophers are nonscientists.

F. If "Some saints were martyrs" is true, what may be inferred about the truth or falsehood of the following propositions? That is, which can be known to be true, which can be known to be false, and which are undetermined?

- \*1. All saints were martyrs.
- 2. All saints were nonmartyrs.
- 3. Some martyrs were saints.
- 4. No saints were martyrs.
- \*5. All martyrs were nonsaints.
- 6. Some nonmartyrs were saints.

- 7. Some saints were not nonmartyrs.
- 8. No martyrs were saints.
- 9. Some nonsaints were martyrs.
- \*10. Some martyrs were nonsaints.
- 11. Some saints were not martyrs.
- 12. Some martyrs were not saints.
- 13. No saints were nonmartyrs.
- 14. No nonsaints were martyrs.
- \*15. Some martyrs were not nonsaints.

G. If "Some merchants are not pirates" is true, what may be inferred about the truth or falsehood of the following propositions? That is, which can be known to be true, which can be known to be false, and which are undetermined?

- \*1. No pirates are merchants.
- 2. No merchants are nonpirates.
- 3. Some merchants are nonpirates.
- 4. All nonmerchants are pirates.
- \*5. Some nonmerchants are nonpirates.
- 6. All merchants are pirates.
- 7. No nonmerchants are pirates.
- 8. No pirates are nonmerchants.
- 9. All nonpirates are nonmerchants.
- \*10. Some nonpirates are not nonmerchants.
- 11. Some nonpirates are merchants.
- 12. No nonpirates are merchants.
- 13. Some pirates are merchants.
- 14. No merchants are nonpirates.
- \*15. No merchants are pirates.

## 5.7 Existential Import and the Interpretation of Categorical Propositions

Categorical propositions are the building blocks of arguments, and our aim throughout is to analyze and evaluate arguments. To do this we must be able to diagram and symbolize the A, E, I, and O propositions. But before we can do that we must confront and resolve a deep logical problem—one that has



("No nonsurgeons are nonphysicians"), which is thus no longer problematic but known to be false.

In the very first chapter of this book we noted that a valid argument whose premises are true *must* have a true conclusion, but also that a valid argument whose premises are false *can* have a true conclusion. Thus, from the false premise, "All animals are cats," the true proposition, "Some animals are cats," follows by subalternation. And from the false proposition, "All parents are students," conversion by limitation yields the true proposition, "Some students are parents." Therefore, if a proposition is given to be false, and the question is raised about the truth or falsehood of some *other*, related proposition, the recommended procedure is to begin drawing immediate inferences from either (1) the contradictory of the proposition known to be false, or (2) the problematic proposition itself. The contradictory of a false proposition must be true, and all valid inferences from that will also be true propositions. And if we follow the other course and are able to show that the problematic proposition implies the proposition that is given is false, we know that it must itself be false. Here follows a table in which the forms of immediate inference—conversion, obversion, and contraposition—are fully displayed:

## OVERVIEW

### Immediate Inferences: Conversion, Obversion, Contraposition

Conversion	
Convertend	Converse
A: All <i>S</i> is <i>P</i> .	I: Some <i>P</i> is <i>S</i> . (by limitation)
E: No <i>S</i> is <i>P</i> .	E: No <i>P</i> is <i>S</i> .
I: Some <i>S</i> is <i>P</i> .	I: Some <i>P</i> is <i>S</i> .
O: Some <i>S</i> is not <i>P</i> .	(conversion not valid)
Obversion	
Obvertend	Obverse
A: All <i>S</i> is <i>P</i> .	E: No <i>S</i> is non- <i>P</i> .
E: No <i>S</i> is <i>P</i> .	A: All <i>S</i> is non- <i>P</i> .
I: Some <i>S</i> is <i>P</i> .	O: Some <i>S</i> is not non- <i>P</i> .
O: Some <i>S</i> is not <i>P</i> .	I: Some <i>S</i> is non- <i>P</i> .
Contraposition	
Premise	Contrapositive
A: All <i>S</i> is <i>P</i> .	A: All non- <i>P</i> is non- <i>S</i> .

E: No <i>S</i> is <i>P</i> .	O: Some non- <i>P</i> is not non- <i>S</i> . (by limitation)
I: Some <i>S</i> is <i>P</i> .	(contraposition not valid)
O: Some <i>S</i> is not <i>P</i> .	O: Some non- <i>P</i> is not non- <i>S</i> .

## EXERCISES

A. State the converses of the following propositions, and indicate which of them are equivalent to the given propositions.

- \*1. No people who are considerate of others are reckless drivers who pay no attention to traffic regulations.
2. All graduates of West Point are commissioned officers in the U.S. Army.
3. Some European cars are overpriced and underpowered automobiles.
4. No reptiles are warm-blooded animals.
- \*5. Some professional wrestlers are elderly persons who are incapable of doing an honest day's work.

B. State the obverses of the following propositions.

- \*1. Some college athletes are professionals.
2. No organic compounds are metals.
3. Some clergy are not abstainers.
4. No geniuses are conformists.
- \*5. All objects suitable for boat anchors are objects that weigh at least fifteen pounds.

C. State the contrapositives of the following propositions and indicate which of them are equivalent to the given propositions.

- \*1. All journalists are pessimists.
2. Some soldiers are not officers.
3. All scholars are nondegenerates.
4. All things weighing less than fifty pounds are objects not more than four feet high.
- \*5. Some noncitizens are not nonresidents.

D. If "All socialists are pacifists" is true, what may be inferred about the truth or falsehood of the following propositions? That is, which can be

## D. CONTRAPOSITION

A third type of immediate inference, **contraposition**, can be reduced to the first two, conversion and obversion. To form the **contrapositive** of a given proposition, we replace its subject term with the complement of its predicate term, and we replace its predicate term with the complement of its subject term. Neither the quality nor the quantity of the original proposition is changed, so the contrapositive of an **A** proposition is an **A** proposition, the contrapositive of an **O** proposition is an **O** proposition, and so forth.

For example, the contrapositive of the **A** proposition, "All members are voters," is the **A** proposition, "All nonvoters are nonmembers." These are logically equivalent propositions, as will be evident on reflection. Contraposition is plainly a valid form of immediate inference when applied to **A** propositions. It really introduces nothing new, because we can get from any **A** proposition to its contrapositive by first obverting it, next applying conversion, and then applying obversion again. Beginning with "All *S* is *P*," we obvert it to obtain "No *S* is non-*P*," which converts validly to "No non-*P* is *S*," whose obverse is "All non-*P* is non-*S*." The contrapositive of any **A** proposition is the obverse of the converse of that proposition.

Contraposition is a valid form of immediate inference when applied to **O** propositions also, although its conclusion may be awkward to express. The contrapositive of the **O** proposition, "Some students are not idealists," is the somewhat cumbersome **O** proposition, "Some nonidealists are not nonstudents," which is logically equivalent to its premise. This also can be shown to be the outcome of first obverting, then converting, then obverting again. "Some *S* is not *P*" obverts to "Some *S* is non-*P*," which converts to "Some non-*P* is *S*," which obverts to "Some non-*P* is not non-*S*."

For **I** propositions, however, contraposition is not, in general, a valid form of inference. The true **I** proposition, "Some citizens are nonlegislators," has as its contrapositive the false proposition, "Some legislators are noncitizens." The reason for this invalidity becomes evident when we try to derive the contrapositive of the **I** proposition by successively obverting, converting, and obverting. The obverse of the original **I** proposition, "Some *S* is *P*," is the **O** proposition, "Some *S* is not non-*P*," but (as we saw earlier) the converse of an **O** proposition does not generally follow validly from it.

In the case of **E** propositions, the contrapositive does not follow validly from the original, as can be seen when, if we begin with the true proposition, "No wrestlers are weaklings," we get, as its contrapositive, the obviously false proposition, "No nonweaklings are nonwrestlers." The reason for this invalidity we will see, again, if we attempt to derive it by successive obversion, conversion, and obversion. If we begin with the **E** proposition, "No *S* is *P*," and obvert it, we obtain the **A** proposition, "All *S* is non-*P*," which in general

cannot be validly converted *except by limitation*. If we do then convert it by limitation to obtain "Some non-*P* is *S*," we can obvert this to obtain "Some non-*P* is not non-*S*." This outcome we may call the *contrapositive by limitation*—and this too we will consider further in the next section.

Contraposition by limitation, in which we infer an **O** proposition from an **E** proposition (for example, we infer "Some non-*P* is not non-*S*" from "No *S* is *P*"), has the same peculiarity as conversion by limitation, on which it depends. Because a particular proposition is inferred from a universal proposition, the resulting contrapositive cannot have the *same* meaning, and cannot be logically equivalent to the proposition that was the original premise. On the other hand, the contrapositive of an **A** proposition is an **A** proposition, and the contrapositive of an **O** proposition is an **O** proposition, and in each of these cases the contrapositive and the premise from which it is derived are equivalent.

Contraposition is thus seen to be valid only when applied to **A** and **O** propositions. It is not valid at all for **I** propositions, and it is valid for **E** propositions only by limitation. The complete picture is exhibited in the following table:

## OVERVIEW

Contraposition	
Premise	Contrapositive
A: All <i>S</i> is <i>P</i> .	A: All non- <i>P</i> is non- <i>S</i> .
E: No <i>S</i> is <i>P</i> .	O: Some non- <i>P</i> is not non- <i>S</i> . (by limitation)
I: Some <i>S</i> is <i>P</i> .	(contraposition not valid)
O: Some <i>S</i> is not- <i>P</i> .	O: Some non- <i>P</i> is not non- <i>S</i> .

Questions about the relations between propositions can often be answered by exploring the various immediate inferences that can be drawn from one or the other of them. For example, given that the proposition, "All surgeons are physicians," is true, what can we know about the truth or falsehood of the proposition, "No nonsurgeons are nonphysicians?" Does this problematic proposition—or its contradictory or contrary—follow validly from the one given as true? To answer we proceed as follows: From what we are given, "All surgeons are physicians," we can validly infer its contrapositive, "All nonphysicians are nonsurgeons." From this, using conversion by limitation (valid according to the traditional view), we can derive "Some nonsurgeons are nonphysicians." But this is the contradictory of the proposition in question

## B. CLASSES AND CLASS COMPLEMENTS

To explain other types of immediate inference we must examine more closely the concept of a "class" and explain what is meant by the *complement of a class*. Any class, we have said, is the collection of all objects that have a certain common attribute, which we may refer to as the "class-defining characteristic." The class of all humans is the collection of all things that have the characteristic of being human; its class-defining characteristic is the attribute of being human. The class-defining characteristic need not be a "simple" attribute; any attribute may determine a class. For example, the complex attribute of being left-handed and red-headed and a student determines a class—the class of all left-handed, red-headed students.

Every class has, associated with it, a **complementary class**, or **complement**, which is the collection of all things that do not belong to the original class. The complement of the class of all people is the class of all things that are *not* people. The class-defining characteristic of that complementary class is the (negative) attribute of not being a person. The complement of the class of all people contains no people, but it contains everything else: shoes and ships and sealing wax and cabbages—but no kings, because kings are people. It is often convenient to speak of the complement of the class of all persons as the "class of all nonpersons." The complement of the class designated by the term *S* is then designated by the term *non-S*; we may speak of the term *non-S* as being the complement of the term *S*.\*

The word *complement* is thus used in two senses. In one sense it is the complement of a class, in the other it is the complement of a term. These are different but very closely connected. One term is the (term) complement of another just in case the first term designates the (class) complement of the class designated by the second term.

Note that a class is the (class) complement of its own complement. Likewise, a term is the (term) complement of its own complement. A sort of "double negative" rule is involved here, to avoid strings of "non's" prefixed to a term. Thus, the complement of the term "voter" is "nonvoter," but the complement of "nonvoter" should be written simply as "voter" rather than as "nonnonvoter."

One must be careful not to mistake contrary terms for complementary terms. "Coward" and "hero" are contraries, because no person can be both a coward and a hero. But we must not identify "cowards" with "nonheroes" because not everyone, and certainly not everything, need be one or the other.

\*Sometimes we reason using what is called the *relative complement of a class*, its complement within some other class. For example, within the class of "children of mine" there is a subclass, "daughters of mine," whose relative complement is another subclass, "children of mine who are not daughters," or "sons of mine." But obversions, and other immediate inferences, rely on the absolute complement of classes, as defined above.

Likewise, the complement of the term "winner" is not "loser" but "nonwinner," for although not everything, or even everyone, is either a winner or a loser, absolutely everything is either a winner or a nonwinner.

## C. OBVERSION

**Obversion** is an immediate inference that is easy to explain once the concept of a term complement is understood. To obvert a proposition, we change its quality (affirmative to negative or negative to affirmative) and replace the predicate term with its complement. However, the subject term remains unchanged, and so does the quantity of the proposition being obverted. For example, the A proposition, "All residents are voters," has as its obverse the E proposition, "No residents are nonvoters." These two are logically equivalent propositions, and either may be validly inferred from the other.

Obversion is a valid immediate inference when applied to *any* standard-form categorical proposition:

- The E proposition, "No umpires are partisans," has as its obverse the logically equivalent A proposition, "All umpires are nonpartisans."
- The I proposition, "Some metals are conductors," has as its obverse the O proposition, "Some metals are not nonconductors."
- The O proposition, "Some nations were not belligerents," has as its obverse the I proposition, "Some nations were nonbelligerents."

The proposition serving as premise for the obversion is called the **obvertend**; the conclusion of the inference is called the **obverse**. Every standard-form categorical proposition is logically equivalent to its obverse, so obversion is a valid form of immediate inference for all standard-form categorical propositions. To obtain the obverse of any proposition, we leave the quantity (universal or particular) and the subject term unchanged; we change the quality of the proposition and replace the predicate term with its complement. The following table gives a complete picture of all valid obversions:

## OVERVIEW

Obversions	
Obvertend	Obverse
A: All <i>S</i> is <i>P</i> .	E: No <i>S</i> is non- <i>P</i> .
E: No <i>S</i> is <i>P</i> .	A: All <i>S</i> is non- <i>P</i> .
I: Some <i>S</i> is <i>P</i> .	O: Some <i>S</i> is not non- <i>P</i> .
O: Some <i>S</i> is not <i>P</i> .	I: Some <i>S</i> is non- <i>P</i> .



## EXERCISES

A. If we assume that the first proposition in each of the following sets is true, what can we affirm about the truth or falsehood of the remaining propositions in each set? B. If we assume that the first proposition in each set is false, what can we affirm?

- \*1. a. All successful executives are intelligent people.  
b. No successful executives are intelligent people.  
c. Some successful executives are intelligent people.  
d. Some successful executives are not intelligent people.
2. a. No animals with horns are carnivores.  
b. Some animals with horns are carnivores.  
c. Some animals with horns are not carnivores.  
d. All animals with horns are carnivores.
3. a. Some uranium isotopes are highly unstable substances.  
b. Some uranium isotopes are not highly unstable substances.  
c. All uranium isotopes are highly unstable substances.  
d. No uranium isotopes are highly unstable substances.
4. a. Some college professors are not entertaining lecturers.  
b. All college professors are entertaining lecturers.  
c. No college professors are entertaining lecturers.  
d. Some college professors are entertaining lecturers.

## 5.6 Further Immediate Inferences

Three other important kinds of immediate inference are not associated directly with the square of opposition: *Conversion*, *Obversion*, and *Contraposition*. These we now explain:

## A. CONVERSION

**Conversion** is an inference that proceeds by interchanging the subject and predicate terms of the proposition. "No men are angels" converts to "No angels are men," and these propositions may be validly inferred from one another. Similarly, "Some women are writers" and "Some writers are women" are logically equivalent, and by conversion either can be validly inferred from the other. Conversion is perfectly valid for all E propositions and for all I propositions. One standard-form categorical proposition is therefore said to be the **converse** of another when we derive it by simply interchanging the subject and predicate terms of that other proposition. The proposition from which it is derived is called the **convertend**. Thus, "No idealists are politicians" is the converse of "No politicians are idealists," which is its convertend.

The conversion of an O proposition is not, in general, valid. The O proposition, "Some animals are not dogs," is plainly true; its converse is the proposition, "Some dogs are not animals," which is plainly false. An O proposition and its converse are not, in general, logically equivalent.

The A proposition presents a special problem here. Of course, the converse of an A proposition does not in general follow from its convertend. From "All dogs are animals" we certainly may not infer that "All animals are dogs." Traditional logic recognized this, of course, but asserted, nevertheless, that something *like* conversion was valid for A propositions. On the traditional square of opposition, one could validly infer from the A proposition, "All dogs are animals," its subaltern I proposition, "Some dogs are animals." The A proposition says something about all members of the subject class (dogs), the I proposition makes a more limited claim, about only some of the members of that class. In general, it was held that one could infer "Some S is P" from "All S is P." And, as we saw earlier, an I proposition may be converted validly; if some dogs are animals, then some animals are dogs.

So, if we are given the A proposition that "All dogs are animals," we first infer that "Some dogs are animals" by subalternation, and from that subaltern we can by conversion validly infer that "Some animals are dogs." Hence, by a combination of subalternation and conversion, we advance validly from "All S is P" to "Some P is S." This pattern of inference, called *conversion by limitation* (or *conversion per accidens*) proceeds by interchanging subject and predicate terms and changing the quantity of the proposition from universal to particular. This type of conversion will be considered further in the next section.

In all conversions, the converse of a given proposition contains exactly the same subject and predicate terms as the convertend, their order being reversed, and always has the same quality (of affirmation or denial). A complete picture of this immediate inference as traditionally understood is given by the following table:

## OVERVIEW

Valid Conversions	
Convertend	Converse
A: All S is P.	I: Some P is S. (by limitation)
E: No S is P.	E: No P is S.
I: Some S is P.	I: Some P is S.
O: Some S is not P.	(conversion not valid)

"Some whales are not fishes." This opposition between a universal proposition and its corresponding particular proposition is known as **subalternation**. In any such pair of corresponding propositions, the universal proposition is called the *superaltern*, and the particular is called the *subaltern*.

In subalternation (in the classical analysis), the superaltern implies the truth of the subaltern. Thus, from the universal affirmative, "All birds have feathers," the corresponding particular affirmative, "Some birds have feathers," was held to follow. From the universal negative, "No whales are fishes," the corresponding particular, "Some whales are not fishes," was likewise held to follow. But of course the implication does not hold from the particular to the universal, from the subaltern to the superaltern. From the proposition, "Some animals are cats," it is obvious that we cannot infer that "All animals are cats." And it would be absurd to infer from "Some animals are not cats" that "No animals are cats."

### E. THE SQUARE OF OPPOSITION

There are thus four ways in which propositions may be "opposed"—as *contradictories*, *contraries*, *subcontraries*, and as *sub-* and *superalterns*. These are represented using an important and widely used diagram called the **square of opposition**, which is reproduced as Figure 5-1.

Relations exhibited by this square of opposition were believed to provide the logical basis for validating certain elementary forms of argument. To explain these, we must first distinguish between **immediate inferences** and **mediate inferences**.

When we draw a conclusion from one or more premises, some inference must be involved. That inference is said to be *mediate* when more than one

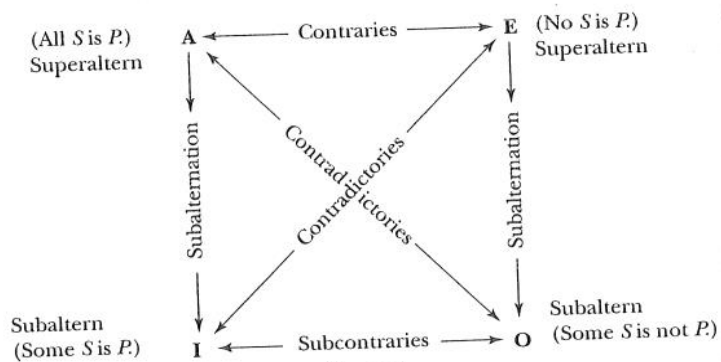


Figure 5-1

premise is relied on (as in a syllogism), because the conclusion is drawn from the first premise through the mediation of the second. However, when a conclusion is drawn from only one premise there is no such mediation, and the inference is said to be *immediate*.

A number of very useful immediate inferences may be readily drawn from the information embedded in the traditional square of opposition. Here are some examples:

- If an A proposition is the premise, then (according to the square of opposition) one can validly infer that the corresponding O proposition (that is, the O proposition with the same subject and predicate terms) is false.
- If an A proposition is the premise, then the corresponding I proposition is true.
- If an I proposition is the premise, its corresponding E proposition, which contradicts it, must be false.

Given the truth, or the falsehood, of any one of the four standard-form categorical propositions, the truth or falsehood of some or all of the others can be inferred immediately. A considerable number of immediate inferences are based on the traditional square of opposition; we list them here:

A is given as true:	E is false; I is true; O is false.
E is given as true:	A is false; I is false; O is true.
I is given as true:	E is false; A and O are undetermined.
O is given as true:	A is false; E and I are undetermined.
A is given as false:	O is true; E and I are undetermined.
E is given as false:	I is true; A and O are undetermined.
I is given as false:	A is false; E is true; O is true.
O is given as false:	A is true; E is false; I is true.*

\*A proposition is undermined if its truth or falsity is not determined—fixed—by the truth or falsity of any other proposition. In another sense, a proposition is undetermined if one does not know that it is true and one also does not know that it is false. If it is given that an A proposition is undetermined, in either sense, we may infer that its contradictory O proposition must be undetermined in that same sense. For if that O proposition were known to be true, the A proposition contradicting it would be known to be false; and if that O proposition were known to be false, the A proposition contradicting it would be known to be true. The same reasoning applies to the other standard-form propositions. In general, if any of the four categorical propositions is given as undetermined in either sense, its contradictory must be undetermined in the same sense.

disagreement between the propositions. The various kinds of opposition are correlated with some very important truth relations, as follows.

### A. CONTRADICTORIES

Two propositions are **contradictories** if one is the denial or negation of the other—that is, if they cannot both be true and cannot both be false. Two standard-form categorical propositions that have the same subject and predicate terms but differ from each other in *both* quantity and quality are contradictories.

Thus the **A** proposition, “All judges are lawyers,” and the **O** proposition, “Some judges are not lawyers,” are clearly contradictories. They are opposed in both quality (one affirms, the other denies) and quantity (one refers to all, and the other to some). Of the pair, exactly one is true and exactly one is false. They cannot both be true; they cannot both be false.

Similarly, the **E** proposition, “No politicians are idealists,” and the **I** proposition, “Some politicians are idealists,” are opposed in both quantity and quality, and they too are contradictories.

In summary: **A** and **O** propositions are contradictories: “All *S* is *P*” is contradicted by “Some *S* is not *P*.” **E** and **I** propositions are also contradictories: “No *S* is *P*” is contradicted by “Some *S* is *P*.”

### B. CONTRARIES

Two propositions are said to be **contraries** if they cannot both be true—that is, if the truth of one entails the falsity of the other. Thus, “Texas will win the coming game with Oklahoma,” and “Oklahoma will win the coming game with Texas,” are contraries. If either of these propositions (referring to the same game, of course) is true, then the other must be false. But these two propositions are not contradictories, because a game could be a draw and then both would be false. Contraries cannot both be true, but, unlike contradictories, they can both be false.

The traditional account of categorical propositions held that universal propositions (**A** and **E**) having the same subject and predicate terms but differing in quality (one affirming, the other denying) were contraries. Thus it was said that an **A** proposition, “All poets are dreamers,” and its corresponding **E** proposition, “No poets are dreamers,” cannot both be true—but they can both be false and may be regarded as contraries.\*

\*This Aristotelian interpretation has some troubling consequences that will be discussed in Section 5.7.

One difficulty with this Aristotelian account arises if either the **A** proposition or the **E** proposition is necessarily true—that is, if either is a logical or mathematical truth, such as “All squares are rectangles,” or “No squares are circles.” In such a case, the claim that the **A** proposition and the **E** proposition are contraries cannot be correct, because a necessarily true proposition cannot possibly be false and so cannot have a contrary, because contraries are two propositions that can both be false. Propositions that are neither necessarily true nor necessarily false are said to be **contingent**. So the reply to this difficulty is that the present interpretation assumes (not unreasonably) that the propositions in question are contingent, in which case the claim that **A** and **E** propositions having the same subject and predicate terms are contraries may be correct. For the remainder of this chapter, we therefore make the assumption that the propositions involved are contingent.

### C. SUBCONTRARIES

Two propositions are said to be **subcontraries** if they cannot both be false, although they may both be true.

The traditional account held that particular propositions (**I** and **O**) having the same subject and predicate terms but differing in quality (one affirming the other denying) are subcontraries. It was said that the **I** proposition, “Some diamonds are precious stones,” and the **O** proposition, “Some diamonds are not precious stones,” could both be true—but they could not both be false and therefore must be regarded as subcontraries.

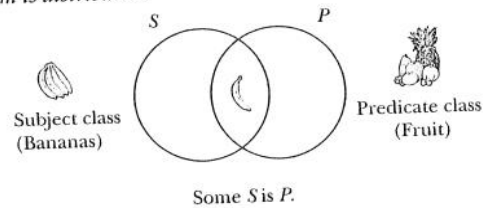
A difficulty similar to the one noted above arises here too. If either the **I** or the **O** proposition is necessarily false (for example, “Some squares are circles” or “Some squares are not rectangles”), it cannot have a subcontrary, because subcontraries are two propositions that can both be true. But if both the **I** and the **O** are contingent propositions, they can both be true, and as we noted in connection with contraries just above, we shall assume for the remainder of this chapter that they are contingent.

### D. SUBALTERNATION

When two propositions have the same subject and the same predicate terms, and agree in quality (both affirming or both denying) but differ in quantity (one universal, the other particular), they are called *corresponding propositions*. This is also a form of “opposition” as that term has traditionally been used. Thus the **A** proposition, “All spiders are eight-legged animals,” has a corresponding **I** proposition, “Some spiders are eight-legged animals.” Likewise, the **E** proposition, “No whales are fishes,” has a corresponding **O** proposition,

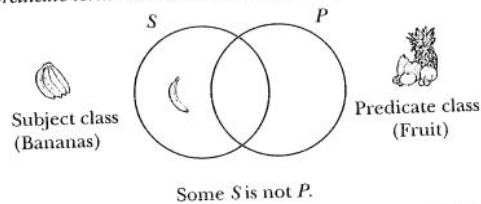


claim about the subject class as a whole. Therefore, in this proposition, as in every I proposition, the subject term is not distributed. Nor does this proposition say anything about every member for the class of fruits (we are told only that there is at least one member of the class of bananas in it), so the predicate is not distributed either. In an I proposition, neither the subject term nor the predicate term is distributed.



### The O proposition: Some bananas are not fruits.

The word "some" again tells us that this proposition is not about all members of the class of bananas; the *subject* term is therefore not distributed. But because we are told, in this proposition, that some bananas are not fruits, we are told something about the entire predicate class—namely, that the entire class of fruits does not have one of those subject bananas among them. In an O proposition, the predicate term is distributed but the subject term is not distributed.



We conclude this section with a table that presents all the critical information about each of the four standard-form categorical propositions:

## OVERVIEW

Proposition	Quantity, Quality, and Distribution			
	Letter Name	Quantity	Quality	Distributes
All S is P.	A	Universal	Affirmative	S only
No S is P.	E	Universal	Negative	S and P
Some S is P.	I	Particular	Affirmative	Neither
Some S is not P.	O	Particular	Negative	P only

## EXERCISES

Name the quality and quantity of each of the following propositions, and state whether their subject and predicate terms are distributed or undistributed.

- \*1. Some presidential candidates will be sadly disappointed people.
2. All those who died in Nazi concentration camps were victims of a cruel and irrational tyranny.
3. Some recently identified unstable elements were not entirely accidental discoveries.
4. Some members of the military-industrial complex are mild-mannered people to whom violence is abhorrent.
- \*5. No leader of the feminist movement is a major business executive.
6. All hard-line advocates of law and order at any cost are people who will be remembered, if at all, only for having failed to understand the major social pressures of the twenty-first century.
7. Some recent rulings of the Supreme Court were politically motivated decisions that flouted the entire history of U.S. legal practice.
8. No harmful pesticides or chemical defoliants were genuine contributions to the long-range agricultural goals of the nation.
9. Some advocates of major political, social, and economic reforms are not responsible people who have a stake in maintaining the status quo.
- \*10. All new labor-saving devices are major threats to the trade union movement.

## 5.5 The Traditional Square of Opposition

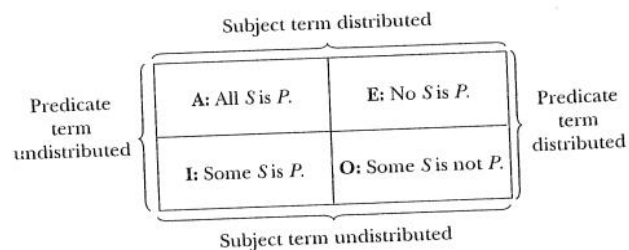
The preceding analysis of categorical propositions enables us to exhibit the relations among those propositions, which in turn provide solid grounds for a great deal of the reasoning we do in everyday life. We need one more technical term: *opposition*. Standard-form categorical propositions having the same subject terms and the same predicate terms may (obviously) differ from each other in quality, or in quantity, or in both. Any such kind of differing has been traditionally called **opposition**. This term is used even when there is no apparent

designated by the subject term; it says that of this part of the class of horses that it is excluded from the class of all thoroughbreds. But they are excluded from the *whole* of the latter class. Given the particular horses referred to, the proposition says that each and every member of the class of thoroughbreds is *not* one of those particular horses. When something is said to be excluded from a class, the whole of the class is referred to, just as, when a person is excluded from a country, all parts of that country are forbidden to that person. In O propositions (particular negatives) the subject term is not distributed, but the predicate term is distributed.

We thus see that universal propositions, both affirmative and negative, distribute their subject terms, whereas particular propositions, whether affirmative or negative, do not distribute their subject terms. Thus the *quantity* of any standard-form categorical proposition determines whether its *subject* term is distributed or undistributed. We likewise see that affirmative propositions, whether universal or particular, do not distribute their predicate terms, whereas negative propositions, both universal and particular, do distribute their predicate terms. Thus the *quality* of a standard-form categorical proposition determines whether its predicate term is distributed or undistributed.

In summary, the A proposition distributes only its subject term; the E proposition distributes both its subject and predicate terms; and the I proposition distributes neither its subject nor its predicate term; and the O proposition distributes only its predicate term.

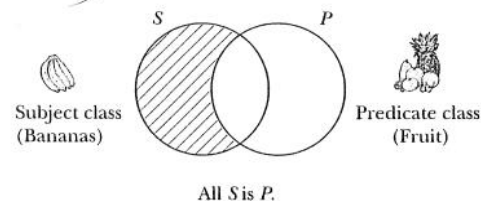
Which terms are distributed by which standard-form categorical propositions will become very important when we turn to the evaluation of syllogisms. The following diagram presents all these distributions graphically, and may be useful in helping you to remember which propositions distribute which of their terms.



## VISUAL LOGIC

## The A proposition: All bananas are fruits.

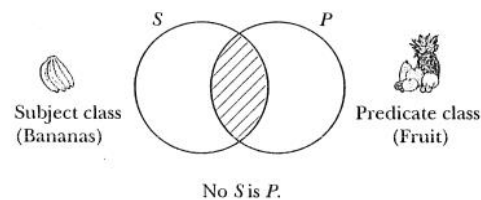
This A proposition asserts that *every* member of the class of bananas (the subject class) is also a member of the class of fruits (the predicate class). When a term refers to every member of a class, we say that it is *distributed*. In an A proposition, the subject term is *always* distributed. But the A proposition does not refer to every member of the predicate class; this example does not assert that all fruits are bananas; it says nothing about every fruit. In an A proposition, the predicate term is *not* distributed.



## The E proposition: No bananas are fruits.

This E proposition asserts that *every* member of the class of bananas is *outside* the class of fruits. The *subject* term, "bananas," is plainly distributed. But because bananas are excluded from the entire class of fruits, this proposition refers to every member of the *predicate* class as well, because it plainly says that *no* fruit is a banana. In an E proposition, both the subject term and the predicate term are distributed.

Note that the concept of distribution has nothing to do with truth or falsity. This example proposition is certainly false—but, as in every E proposition, both of its terms are distributed.



## The I proposition: Some bananas are fruits.

The word "some" in this I proposition tells us that at least one member of the class designated by the subject term, "bananas," is also a member of the class designated by the predicate term, "fruits"—but this proposition makes no

(Continued)

whether complete or partial, its quality is *negative*. So the E proposition, "No S is P," and the O proposition, "Some S is not P," are both negative in quality. Their letter names, E and O, are thought to come from the Latin word, "nEgO," meaning "I deny." Every categorical proposition has one quality or the other, affirmative or negative.

## B. QUANTITY

Every standard-form categorical proposition has some class as its subject. If the proposition refers to all members of the class designated by its subject term, its quantity is *universal*. So the A proposition, "All S is P," and the E proposition, "No S is P," are both universal in quantity. If the proposition refers only to some members of the class designated by its subject term, its quantity is *particular*. So the I proposition, "Some S is P," and the O proposition, "Some S is not P," are both particular in quantity.

The quantity of a standard-form categorical proposition is revealed by the word with which it begins, either "all," "no," or "some." "All" and "no" indicate that the proposition is universal; "some" indicates that the proposition is particular. The word "no" serves also, in the case of the E proposition, to indicate its negative quality, as we have seen.

Because every standard-form categorical proposition must be either affirmative or negative, and must be either universal or particular, the four names uniquely describe each one of the four standard forms by indicating its quantity and its quality: universal affirmative (A), particular affirmative (I), universal negative (E), particular negative (O).

## C. GENERAL SCHEMA OF STANDARD-FORM CATEGORICAL PROPOSITIONS

Between the subject and predicate terms of every standard-form categorical proposition occurs some form of the verb "to be." This verb (accompanied by "not" in the case of the O proposition) serves to connect the subject and predicate terms and is called the *copula*. Writing the four propositions schematically, as we did earlier (All S is P, Some S is P, etc.), only the words "is" and "is not" appear; but (depending on context) other forms of the verb "to be" may be appropriate. We may change the tense (for example, "Some Roman emperors were monsters" or "Some soldiers will not be heroes"), or change to the plural form of the verb (for example, "All squares are rectangles"). In these examples, "were" and "are" and "will not be" serve as copulas. However, the general skeleton of a standard-form categorical proposition always consists of just four parts: first the quantifier, then the

subject term, next the copula, and finally the predicate term. The schema may be written as

Quantifier (subject term) copula (predicate term).

## D. DISTRIBUTION

Categorical propositions are regarded as being about classes, the classes of objects designated by the subject and predicate terms. We have seen that a proposition may refer to classes in different ways; it may refer to *all* members of a class or refer to only *some* members of that class. Thus the proposition, "All senators are citizens," refers to, or is about, *all* senators, but it does not refer to all citizens. That proposition does not affirm that every citizen is a senator, but it does not deny it either. Every A proposition is thus seen to refer to all members of the class designated by its subject term, S, but does *not* refer to all members of the class designated by its predicate term, P.

To characterize the ways in which terms can occur in categorical propositions, we introduce the technical term **distribution**. A proposition distributes a term if it refers to all members of the class designated by that term. In A, E, I, and O propositions, the terms that are distributed vary, as follows.

**In the A proposition (e.g., "All senators are citizens"):** In this proposition, "senators" is distributed, but "citizens" is not. In A propositions (universal affirmatives) the subject term is distributed, but the predicate term is undistributed.

**In the E proposition (e.g., "No athletes are vegetarians"):** The subject term, "athletes," is distributed, because the whole class of athletes is said to be excluded from the class of vegetarians. But in asserting that the whole class of athletes is excluded from the class of vegetarians, it is also asserted that the whole class of vegetarians is excluded from the class of athletes. Of each and every vegetarian, the proposition says that he or she is not an athlete. Unlike an A proposition, therefore, an E proposition refers to all members of the class designated by its predicate term, and therefore also distributes its predicate term. E propositions (universal negatives) distribute both their subject and their predicate terms.

**In the I proposition (e.g., "Some soldiers are cowards"):** No assertion is made about all soldiers in this proposition, and no assertion is made about all cowards either. It says nothing about each and every soldier, and nothing about each and every coward. Neither class is wholly included, or wholly excluded, from the other. In I propositions (particular affirmatives) both subject and predicate terms are not distributed.

**In the O proposition (e.g., "Some horses are not thoroughbreds"):** Nothing is said about all horses. The proposition refers to some members of the class



This analysis of categorical propositions appears to be simple and straightforward, but the discovery of the fundamental role of these propositions, and the exhibition of their relations to one another, was a great step in the systematic development of logic. It was one of Aristotle's permanent contributions to human knowledge. Its apparent simplicity is deceptive. On this foundation—classes of objects and the relations among those classes—logicians have erected, over the course of centuries, a highly sophisticated system for the analysis of deductive argument. This system, whose subtlety and penetration mark it as one of the greatest of intellectual achievements, we now explore in the following three steps:

- A. In the remainder of this chapter we examine the features of standard-form categorical propositions more deeply, explaining their relations to one another. We show what inferences may be drawn *directly* from these categorical propositions. A good deal of deductive reasoning, we will see, can be mastered with no more than a thorough grasp of A, E, I, and O propositions and their interconnections.
- B. In the next chapter, we explain *sylogisms*—the arguments that are commonly constructed using standard-form categorical propositions. We explore the realm of syllogisms, in which every valid argument form is uniquely characterized and given its own name. And we develop powerful techniques for determining the validity (or invalidity) of syllogisms.
- C. In Chapter 7 we integrate syllogistic reasoning and the language of argument in everyday life. We identify some limitations of reasoning based on this foundation, but we also glimpse the penetration and wide applicability that this foundation makes possible.

## OVERVIEW

Standard-Form Categorical Propositions		
Proposition Form	Name and Type	Example
All <i>S</i> is <i>P</i> .	A Universal affirmative	All lawyers are wealthy people.
No <i>S</i> is <i>P</i> .	E Universal negative	No criminals are good citizens.
Some <i>S</i> is <i>P</i> .	I Particular affirmative	Some chemicals are poisons.
Some <i>S</i> is not <i>P</i> .	O Particular negative	Some insects are not pests.

## EXERCISES

Identify the subject and predicate terms in, and name the form of, each of the following propositions.

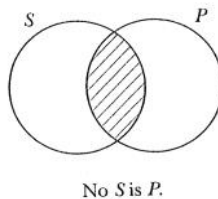
- \*1. Some historians are extremely gifted writers whose works read like first-rate novels.
2. No athletes who have ever accepted pay for participating in sports are amateurs.
3. No dogs that are without pedigrees are candidates for blue ribbons in official dog shows sponsored by the American Kennel Club.
4. All satellites that are currently in orbits less than ten thousand miles high are very delicate devices that cost many thousands of dollars to manufacture.
- \*5. Some members of families that are rich and famous are not persons of either wealth or distinction.
6. Some paintings produced by artists who are universally recognized as masters are not works of genuine merit that either are or deserve to be preserved in museums and made available to the public.
7. All drivers of automobiles that are not safe are desperadoes who threaten the lives of their fellows.
8. Some politicians who could not be elected to the most minor positions are appointed officials in our government today.
9. Some drugs that are very effective when properly administered are not safe remedies that all medicine cabinets should contain.
- \*10. No people who have not themselves done creative work in the arts are responsible critics on whose judgment we can rely.

## 5.4 Quality, Quantity, and Distribution

### A. QUALITY

Every standard-form categorical proposition either affirms, or denies, some class relation, as we have seen. If the proposition affirms some class inclusion, whether complete or partial, its quality is *affirmative*. So the A proposition, "All *S* is *P*," and the I proposition, "Some *S* is *P*," are both affirmative in quality. Their letter names, A and I, are thought to come from the Latin word, "Affirmo," meaning, "I affirm." If the proposition denies class inclusion,

the classes  $S$  and  $P$  shaded out. So the E proposition is diagrammed thus:



3. **Particular affirmative propositions.** The third example above, "Some politicians are liars," affirms that some members of the class of all politicians are members of the class of all liars. But it does not affirm this of politicians universally. Only some particular politician or politicians are said to be liars. This proposition does not affirm or deny anything about the class of all politicians; it makes no pronouncements about that entire class. Nor does it say that some politicians are not liars, although in some contexts it may be taken to suggest that. The literal and exact interpretation of this proposition is the assertion that the class of politicians and the class of liars *have some member or members in common*. That is what we understand this standard form proposition to mean.

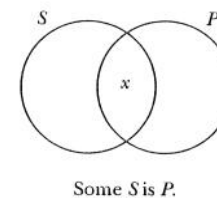
"Some" is an indefinite term. Does it mean "at least one," or "at least two," or "at least several"? Or how many? Context might affect our understanding of the term as it is used in everyday speech, but logicians, for the sake of definiteness, interpret "some" to mean "at least one." A particular affirmative proposition may be written schematically as

Some  $S$  is  $P$ .

which says that at least one member of the class designated by the subject term  $S$  is also a member of the class designated by the predicate term  $P$ . The proposition *affirms* that the relation of class *inclusion* holds, but does not affirm it of the first class universally but only partially, that is, it is affirmed of some *particular* member, or members, of the first class. Propositions in this standard form are called *particular affirmative propositions*. They are also called **I** propositions.

The diagram for the **I** proposition indicates that there is at least one member of  $S$  that is also a member of  $P$  by placing an  $x$  in the

region in which the two circles overlap. So the **I** proposition is diagrammed thus:

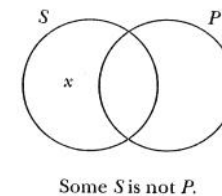


4. **Particular negative propositions.** The fourth example above, "Some politicians are not liars," like the third, does not refer to politicians universally, but only to *some* member or members of that class; it is *particular*. Unlike the third example, however, it does not affirm the inclusion of some member or members of the first class in the second class; this is precisely what is *denied*. It is written schematically as

Some  $S$  is not  $P$ .

which says that at least one member of the class designated by the subject term  $S$  is excluded from the whole of the class designated by the predicate term  $P$ . The denial is not universal. Propositions in this standard form are called *particular negative propositions*. They are also called **O** propositions.

The diagram for the **O** proposition indicates that there is at least one member of  $S$  that is not a member of  $P$  by placing an  $x$  in the region of  $S$  that is outside of  $P$ . So the **O** proposition is diagrammed thus:



The examples we have used in this section employ classes that are simply named: politicians, liars, vegetarians, athletes, and so on. But subject and predicate terms in standard-form propositions can be more complicated. Thus, for example, the proposition "All candidates for the position are persons of honor and integrity" has the phrase "candidates for the position" as its subject term and the phrase "persons of honor and integrity" as its predicate term. Subject and predicate terms can become more intricate still, but in each of the four standard forms a relation is expressed between a subject class and a predicate class. These four—**A**, **E**, **I**, and **O** propositions—are the building blocks of deductive arguments.

Categorical propositions are the fundamental elements, the building blocks of argument, in the classical account of deductive logic. Consider the argument

No athletes are vegetarians.

All football players are athletes.

Therefore no football players are vegetarians.

This argument contains three categorical propositions. We may dispute the truth of its premises, of course, but the relations of the classes expressed in these propositions yields an argument that is certainly valid: If those premises are true, that conclusion *must* be true. And it is plain that each of the premises is indeed categorical; that is, *each premise affirms, or denies, that some class S is included in some other class P, in whole or in part*. In this illustrative argument the three categorical propositions are about the class of all athletes, the class of all vegetarians, and the class of all football players.

The critical first step in developing a theory of deduction based on classes, therefore, is to identify the kinds of categorical propositions and to explore the relations among them.

### 5.3 The Four Kinds of Categorical Propositions

There are four and only four kinds of **standard-form categorical propositions**. Here are examples of each of the four kinds:

1. All politicians are liars.
2. No politicians are liars.
3. Some politicians are liars.
4. Some politicians are not liars.

We will examine each of these kinds in turn.

1. **Universal affirmative propositions.** In these we assert that *the whole of one class is included or contained in another class*. "All politicians are liars" is an example; it asserts that every member of one class, the class of politicians, is a member of another class, the class of liars. Any universal affirmative proposition can be written schematically as

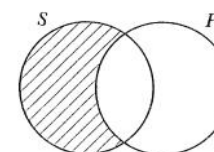
All S is P.

where the letters *S* and *P* represent the *subject* and *predicate* terms, respectively. Such a proposition *affirms* that the relation of class *inclusion*

holds between the two classes and says that the inclusion is complete, or *universal*. All members of *S* are said to be also members of *P*. Propositions in this standard form are called *universal affirmative propositions*. They are also called *A propositions*.

Categorical propositions are often represented with diagrams, using two interlocking circles to stand for the two classes involved. These are called **Venn diagrams**, named after the English logician and mathematician, John Venn (1824–1923), who invented them. Later we will explore these diagrams more fully, and we will find that such diagrams are exceedingly helpful in appraising the validity of deductive arguments. For the present we use these diagrams only to exhibit graphically the sense of each categorical proposition.

We label one circle *S*, for subject class, and the other circle *P*, for predicate class. The diagram for the *A* proposition, which asserts that all *S* is *P*, shows that portion of *S* which is outside of *P* shaded out, indicating that there are no members of *S* that are not members of *P*. So the *A* proposition is diagrammed thus:



All S is P.

2. **Universal negative propositions.** The second example above, "No politicians are liars," is a proposition in which it is denied, universally, that any member of the class of politicians is a member of the class of liars. It asserts that the subject class, *S*, is wholly excluded from the predicate class, *P*. Schematically, categorical propositions of this kind can be written as

No S is P.

where again *S* and *P* represent the subject and predicate terms. This kind of proposition *denies* the relation of *inclusion* between the two terms, and denies it *universally*. It tells us that no members of *S* are members of *P*. Propositions in this standard form are called *universal negative propositions*. They are also called *E propositions*.

The diagram for the *E* proposition will exhibit this mutual exclusion by having the overlapping portion of the two circles representing



## Categorical Propositions

- 5.1 The Theory of Deduction
- 5.2 Classes and Categorical Propositions
- 5.3 The Four Kinds of Categorical Propositions
- 5.4 Quality, Quantity, and Distribution
- 5.5 The Traditional Square of Opposition
- 5.6 Further Immediate Inferences
- 5.7 Existential Import and the Interpretation of Categorical Propositions
- 5.8 Symbolism and Diagrams for Categorical Propositions

### 5.1 The Theory of Deduction

We turn now to analysis of the structure of arguments. Preceding chapters have dealt mainly with the language in which arguments are formulated. In this and succeeding chapters we explore and explain the relations between the premises of an argument and its conclusion.

All of Part II of this book is devoted to **deductive arguments**. A deductive argument is one whose premises are claimed to provide conclusive grounds for the truth of its conclusion. If that claim is correct—that is, if the premises of the argument really do assure the truth of its conclusion with necessity—that deductive argument is **valid**. Every deductive argument either does what it claims, or it does not; therefore, every deductive argument is either valid or invalid. If it is valid, it is impossible for its premises to be true without its conclusion also being true.

The theory of deduction aims to explain the relations of premises and conclusion in valid arguments. It also aims to provide techniques for the appraisal of deductive arguments, that is, for discriminating between valid and invalid deductions. To accomplish this, two large bodies of theory have been developed. The first is called **classical logic** or **Aristotelian logic**, after the Greek philosopher who initiated this study. The second is called **modern logic** or **modern symbolic logic**, developed mainly during the twentieth century. Classical logic is the topic of this and the following two chapters (Chapters 5, 6, and 7); modern symbolic logic is the topic of Chapters 8, 9, and 10.

Aristotle (384–322 B.C.) was one of the towering intellects of the ancient world. After studying for twenty years in Plato's Academy, he became tutor to Alexander the Great; later he founded his own school, the Lyceum, where he contributed substantially to nearly every field of human knowledge. His great treatises on reasoning were gathered together after his death and came to be called the *Organon*, meaning literally the "instrument," the fundamental tool of knowledge.

The word *logic* did not acquire its modern meaning until the second century A.D., but the subject matter of logic was long understood to be the matters treated in Aristotle's seminal *Organon*. Aristotelian logic has been the foundation of rational analysis for thousands of years. Over the course of those centuries it has been very greatly refined: its notation has been much improved, its principles have been carefully formulated, its intricate structure has been completed. This great system of classical logic, set forth in this and the next two chapters, remains an intellectual tool of enormous power, as beautiful as it is penetrating.

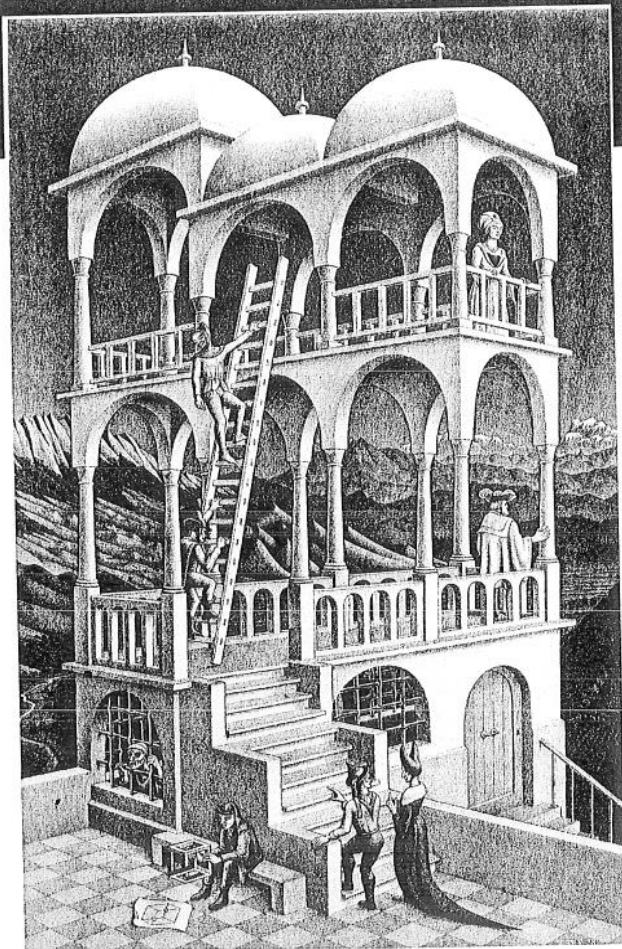
### 5.2 Classes and Categorical Propositions

Classical logic deals mainly with arguments based on the relations of classes of objects to one another. By a **class** we mean a collection of all objects that have some specified characteristic in common.\* Everyone can see immediately that two classes can be related in at least the following three ways:

1. All of one class may be included in all of another class. Thus the class of all dogs is *wholly included* (or wholly contained) in the class of all mammals.
2. Some, but not all, of the members of one class may be included in another class. Thus the class of all athletes is *partially included* (or partially contained) in the class of all females.
3. Two classes may have no members in common. Thus the class of all triangles and the class of all circles may be said to *exclude* one another.

These three relations may be applied to classes, or categories, of every sort. In a deductive argument we present propositions that state the relations between one category and some other category. The propositions with which such arguments are formulated are therefore called **categorical propositions**.

\*The concept of classes was introduced briefly in Chapter 3, in explaining definitions based on the intension of terms.

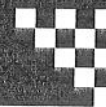


*Belvedere*, by M.C. Escher, depicts a structure in which the relations of the base to the middle and upper portions are not rational; the pillars seem to connect the parts, but do so in ways that make no sense when closely examined. One pillar, resting on the railing at the rear, appears to support the upper story in front; two other pillars, which rise from the balustrade at the top of the front staircase, appear to support the upper portion of the building at its very rear! No such structure could ever stand.

A deductive argument rests upon premises that serve as its foundation. To succeed, its parts

must be held firmly in place by the reasoning that connects those premises to all that is built upon them. If the deductive inferences are solid and reliable at every point, the argument may stand. But if any proposition in the argument is asserted on the basis of other propositions that cannot bear its weight, the argument will collapse as *Belvedere* would collapse. The architect studies the links that can make a building secure; the logician studies the links that can make a deductive argument valid.

M. C. Escher's *Belvedere* © 2004 The M.C. Escher Company. Baam, Holland. All rights reserved.



## PART II

### Deduction

#### SECTION A CLASSICAL LOGIC

CHAPTER 5 Categorical Propositions

CHAPTER 6 Categorical Syllogisms

CHAPTER 7 Syllogisms in Ordinary Language

#### SECTION B MODERN LOGIC

CHAPTER 8 Symbolic Logic

CHAPTER 9 Methods of Deduction

CHAPTER 10 Quantification Theory

*For as one may feel sure that a chain will hold when he is assured that each separate link is of good material and that it clasps the two neighboring links, namely, the one preceding and the one following it, so we may be sure of the accuracy of the reasoning when the matter is good, that is to say, when nothing doubtful enters into it, and when the form consists in a perpetual concatenation of truths which allows of no gap.*

—Gottfried Leibniz